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*Photodynamic Notes, V. By Pliny Earle Chase, LL.D.**(Read before the American Philosophical Society, April 21st, 1882.)*158. *Synchronous Areas.*

Kepler's second law is grounded upon principles which must modify rotation and subsidence, so as to introduce harmonic tendencies among the synchronous areas which are described by different bodies, under the controlling activity of a common centre, as well as in the virtual areas which represent the reaction of the subordinate masses upon the centre of gravity of the system. In orbits of small eccentricity, the instantaneous area of a particle is nearly proportional to the square root of its mean radius vector. If we take $r = (\frac{1}{2})^2 = .125$, as a harmonic divisor, the first of these tendencies is shown by the principal planets, as may be seen in the following table :

Harmonic Areas.		Synchronous Areas.		Difference.
5 r	.625	Mercury	.6223	+ .0028
7 r	.875	Venus,	.8505	+ .0245
8 r	1.000	Earth,	1.0000	.0000
10 r	1.250	Mars,	1.2344	+ .0156
18 r	2.250	Jupiter,	2.2910	— .0310
25 r	3.125	Saturn,	3.0885	+ .0365
35 r	4.375	Uranus,	4.3799	— .0049
44 r	5.500	Neptune,	5.4803	+ .0197

All the differences are within the limits of probable error, .03125, except Saturn's. Jupiter's area is nearly $\frac{3}{4}$ of Saturn's, and the combined masses of these two planets is so great as partially to override the simple tendencies of subsidence towards the chief centres of condensation and nucleation, Earth and Sun.

The synchronous areas of Mercury and Mars, the outliers of the dense belt, are nearly in the ratio 1 : 2; Venus and Earth, 7 : 8; Uranus and Neptune, 4 : 5. The difference is less than $\frac{1}{46}$ of the probable error in the first of these comparisons; less than $\frac{1}{2}$ of the probable error in the second; less than $\frac{1}{109}$ of the probable error in the third; the "probable error," in each case, being $\frac{1}{4}$ of the common divisor, or the deviation which would be admissible without weakening the evidence of harmonic tendency in a *vera causa*.

159. *Virtual Areas.*

The virtual areas of synchronous reaction, or the instantaneous areas which a particle, at Sun's mean distance, would describe about the principal planets if it were not restrained by stronger influences, vary as $\sqrt{m} r$. *Vis viva* may be represented by orbital areas, as well as by distances of projection against uniform resistance, therefore we may add a third law to

Laplace's two laws of constant sums, viz :—*The sum of all the instantaneous virtual areas in a system will always remain invariable.*

From Alexander's harmony (Note 156, p. 605) it follows, that the ratio between the virtual areas of Jupiter and Saturn is nearly the reciprocal of the ratio of their direct areas. The harmonic influence of the repeated nodal action of this ratio, upon subordinate planetary aggregation, is shown in the following table :

Harmonic Areas.		Virtual Areas.		Difference.
α	40.256	Jupiter,	40.587	— .331
$\beta = \frac{3}{4} \alpha$	30.192	Saturn,	30.063	+ .129
$\gamma = \frac{3}{4} \beta$	22.644	Neptune,	22.675	— .031
$\delta = \frac{3}{4} \gamma$	16.983	Uranus,	16.782	+ .201
ε	1.000	Earth,	1.000	.000
$\zeta = \frac{3}{4} \varepsilon$.750	Venus,	.749	+ .001
$\eta = \frac{2}{5} \varepsilon$.400	Mars,	.404	— .004

The greatest proportionate difference is that of Uranus, $1\frac{1}{5}$ per cent. The harmonic change from the outer to the inner belt of planets, $\delta \div \varepsilon = 16.983$, represents the orbital retardation at the chief centre of condensation, Earth. If Earth were rotating with the speed which it would have if Laplace's limit coincided with its equatorial surface, its time of rotation would be $2\pi \sqrt{\frac{r}{g}} = 5073.6$ seconds ; $86164.1 \div 5073.6 = 16.983$. The synchronous virtual area of Mars differs by less than $4\frac{1}{2}$ per cent. from $\frac{3}{4}$ of $\frac{3}{4} \zeta$. This is less than 18 per cent. of the probable error.

160. Laplace's First Law of Stability.

The first of the two laws in which the author of *Mécanique Céleste* embodied his discoveries in relation to the stability of the solar system, is thus stated ; "If the mass of each planet be multiplied by the product of the square of the eccentricity and square root of the mean distance, the sum of all these products will always retain the same magnitude." By combining the first and third of these factors, $m \sqrt{r}$, we get the quotient of mass by orbital velocity, together with the following suggestions of nodal influence:

	$m\sqrt{r}$	Semi-axes major.
Jupiter	$722.19 = 5.184^4$	γ_5 5.203
Saturn,	$279.46 = 9.695^{\frac{5}{2}}$	γ_6 9.539
Neptune,	$93.82 = 30.146^{\frac{4}{3}}$	γ_9 30.037
Earth,	1.00	γ_3 1.000

Jupiter's exponent represents the variable ratio of subsidence-acceleration to orbital velocity ; Saturn's the product of orbital time by mean distance ; Neptune's, the variable ratio of Laplace's limit to nucleal radius.

161. *Orbital Momentum.*

The division of $m \sqrt{r}$ by r gives the product of mass by orbital velocity, or orbital momentum, together with the following suggestions of photodynamic or nebular activity :

	$m \div \sqrt{r}$	Cardinal Radii.	
Jupiter,	$138.81 = 5.178^3$	γ_5	5.203
Saturn,	$30.68 = 9.800^{\frac{3}{2}}$	β_6	10.000
Uranus,	$3.35 = 20.567^{\frac{2}{3}}$	α_7	20.679
Neptune,	$3.12 = 30.483^{\frac{1}{3}}$	α_8	30.470
Earth,	1.00	γ_3	1.000

Jupiter's exponent represents the ratio of its photodynamic orbital volume to that of Earth ; Saturn's, the ratio of orbital times ; Uranus's the influence of mean rotary *vis viva* in an elastic medium ; Neptune's the influence of a centre of linear oscillation in an elastic medium.

162. *Coefficient of Solar Torsion.*

In applying the oscillatory equation, $t = \pi \sqrt{\frac{l}{g}}$, at the centre of gravity of a stellar system, let t represent the duration of an oscillation or half-rotation, g the acceleration of gravity at the stellar equatorial surface, $\pi^2 l$ the stellar modulus of light or the height of a homogeneous æthereal atmosphere which would propagate undulations with the velocity of light. Then, if the stellar rotary oscillation is due to the reaction of cosmical inertia against æthereal influence, gt is equivalent to the velocity of light, v_λ .

In Coulomb's formula of torsional elasticity, $f = \frac{\pi^2 a^2 W}{2 gt^2}$, W represents a weight suspended by a wire, a the coefficient of the radius of torsion, f the coefficient of torsion for the extended wire, g gravitating acceleration, t time of oscillation when the force of torsion is removed. Applying this formula to solar rotation, we have

$$f = \frac{m}{2} \cdot \frac{W}{2} \cdot \frac{\pi^2 a^2 r_o}{gt^2}; \therefore \pi^2 a^2 r_o = gt^2 = \pi^2 l.$$

But gt is the velocity which would be communicated by gravity, at Sun's surface, in one oscillation of half-rotation, or the velocity of light ; gt^2 is the modulus of light at Sun's surface ; $a^2 r_o$ is the theoretical length of a pendulum, at Sun's surface, which would oscillate once in each half-rotation ; $a r_o$ is the length of an equatorial radius rotating with Sun and having the superficial orbital velocity, \sqrt{gr} , at its remote extremity. These are the same results as have been already derived from simple gravitating and radiodynamic considerations, Notes 17, 48, 100, etc. Their statement in this form may be satisfactory to some readers who have not followed the foregoing investigations through all their details.

163. *Harmonic Categories.*

The simple discovery of so many harmonies, in all departments of physical science, would be interesting, even if it were accidental or wholly empirical. The fact that the discovery has sprung from systematic investigations, under the guidance of well-known laws, adds much to its importance. The following results seem to be especially important, and somewhat typical.

1. The equality of gt , in the solar oscillations of half-rotation, to the *velocity of light*. Notes 17, 162, etc.

2. The relations of mass and *vis viva* which satisfy cosmical tendencies to nodality, subsidence, oscillation and orbital revolution. Notes 5, 23, 79, 91, 156, 158-61.

3. The far-reaching evidence of elastic influence which establishes measurable progressive relations between the solar system and the fixed stars. Notes 46, 111-5, 130-2, 155.

4. The simplicity of the relations between elastic and cosmical *vis viva*, which furnish data for approximate estimates of Sun's mass and distance by means of barometric fluctuations. Notes 104-5.

5. The relations of magnetic and cosmical *vis viva*, together with the evidence which they furnish of the dependence of solar and lunar magnetic disturbances upon thermal and tidal influences. Notes 2, 116-22, 125-6.

6. The curiously symmetric harmony in Mars and its satellite-system. Note 28.

7. The varied harmonies of spectral lines, together with the relations of planetary positions to luminous nodes. Notes 36-45, 109, 141-2, 144-53, 157.

8. The confirmations of predictions which were founded upon evidences of the influence of harmonic laws. Notes 33, 133, etc.

9. The interchangeable convertibility of physical units. Notes 90, 96.

10. Atomic phyllotaxy. Notes 135-9, 143. Although Gerber's divisors were found empirically, they represent natural elementary groups. His utter want of suspicion that they had any physical meaning makes them much more important than they would have been if his investigations had been biased by a preconceived hypothesis. The kinetic theory of gases necessitates harmonic action, and the tendency to division in extreme and mean ratio leads to one of the most simple kinds of harmony. There is no necessary inconsistency between the doctrine of atomic phyllotaxy and Prout's hypothesis.

164. *Mercury's Virtual Area.*

The fundamental ratio of successive virtual areas, $\frac{3}{2}$, represents the ratio of the locus of linear centre of gravity of a simple pendulum to the locus of its centre of oscillation, as well as the exponential ratio of nucleation to limitation in an elastic medium. The intermediate step between the harmonic areas for Mars and Venus, Note 159, may, perhaps, be distributed,

partly among the asteroids, partly in satisfying special requirements of the dense belt, and partly in the variations of æthereal *vis viva*. The mass of Mercury is so imperfectly known that it is unsafe to put much trust in the accuracy of any merely harmonic indications of its value, but its virtual area is unquestionably of the same order of magnitude as $(\frac{3}{4})^3$ of that of Mars, or $\frac{7}{10}\frac{2}{9}\frac{9}{5}$ of that of Earth. This would give, for an approximate estimate of the quotient of Sun's mass by that of Mercury, 4054440. The two intermediate steps may, perhaps, be partly absorbed by the intra-Mercurial harmonic nodes and the meteoroids of the zodiacal light.

165. *Relative Masses of Neptune and Mars.*

An intermediate step between the virtual areas (Note 159) and the nodal masses (Note 156), is indicated by the ratio between the masses at the outer limits of the supra-asteroidal and the intra-asteroidal belts. The quotient of the square of Neptune's harmonic virtual area, 22.644², by its harmonic radius, 30.036*, is 17.071; the quotient of the squared area of Mars, $(\frac{3}{4})^6$, by its harmonic radius, 1.669, is .10664; the ratio of the masses and the mass-ratio of Sun to Mars are approximately shown in the following proportions:

$$\begin{aligned} m_8 : m_4 &:: 17.071 : .10664 :: 160.09 : 1 \\ m_6 : m_1 &:: (160.09 \times 19380 = 3102544) : 1 \end{aligned}$$

166. *Various Harmonic Indications and Tests.*

If κ represents Earth's limiting nucleal radius (Note 159), the corresponding atmospheric radius would be $\kappa^{\frac{4}{3}} = 43.653$. Herschel's locus of incipient subsidence, in the controlling two-planet belt, or Saturn's secular aphelion, is 1.0843289 times the outer limiting locus of the belt (Stockwell, *Smithson. Contrib.*, 232, p. 38); $\kappa^{\frac{4}{3}} \div 1.0843289 = 40.258$, which is, with close approximation, the ratio of the instantaneous virtual area at the inner locus of the controlling belt, to the corresponding area at the chief centre of condensation. The tendency of exponents, in elastic media, to become coefficients of elastic *vis viva*, is shown in Note 159. If we use the symmetrical harmonic areas for Mars and Mercury, the percentages of difference between the harmonic and virtual areas are, respectively, $\frac{5}{8}$ of .01, $\frac{3}{7}$ of .01, $\frac{1}{7}$ of .01, $\frac{5}{8}$ of .01, $\frac{1}{7}$ of .01, .045, .099. In testing the combined harmonic influences of a *vera causa* which is subject to internal perturbations, there is room for a possible deviation of 50 per cent. and a probable deviation of 25 per cent. The combined probability that the approximations in Note 159 are owing to æthereal influence is, therefore, $30 \times \frac{1}{3}\frac{5}{8} \times 175 \times \frac{1}{3}\frac{5}{8} \times 175 \times \frac{5}{9} \times \frac{2}{5}\frac{5}{9} = 15664091727 : 1$.

The following points of symmetry and alternation may be noted in the nodal mass-factors of the two outer planets, Note 156:

1. The tendency to equality of mean orbital *vis viva* in Earth, Uranus and Neptune, as indicated by the factors γ_3 , γ_7 and γ_9 .

*Proc. Am. Ph. Soc., xiii, 239.

2. The nodal modification of Neptune's mass by Earth's secular aphelion, and of the mass of Uranus by Earth's secular perihelion.

3. The nodal modification of Neptune's mass by its own mean perihelion, and of the mass of Uranus by its own mean aphelion.

4. The modification of Uranus by Jupiter, and the corresponding modification of Neptune by Uranus.

167. *Earth's Modulus of Rotation and Jupiter's Eccentricity.*

Let g_β represent the sum of the gravitating accelerations of Sun and Earth at Earth's equatorial surface; t , time of Earth's rotary oscillation ($\frac{1}{2}$ sidereal day); ρ_0 , Sun's equatorial semi-diameter; r_3 , Earth's semi-diameter; ρ_a , mean projection of centre of gravity of Sun and Jupiter from ρ_0 ; $\rho_a \div \rho_0$, Jupiter's maximum secular eccentricity; ρ_3 , Earth's semi-axis major; $g_\beta t^2$, Earth's modulus of rotation. Then

$$g_\beta t^2 : \rho_3 :: \rho_a : \rho_0$$

The photodynamic or oscillatory values of Sun's mass and distance, Note 91, give for Sun's gravitating acceleration of Earth $\frac{m_0}{m_3} \left(\frac{r_3}{\rho_3} \right)^2 = 331776 \times (3962.8 \div 92785700)^2 = .000605184$ of Earth's equatorial gravitating acceleration. If we adopt Everett's value for g , $g_\beta t^2 = 1.000605184 \times 32.091 \times 43082^2 \div 5280 = 5643840$ miles; $\rho_a \div \rho_0 = .0608265$. Stockwell's value (*Smith. Cont.*, 232, p. 38), is .0608274.

168. *Axis of Central Subsidence and Rupture.*

The influence of the interstellar photodynamic paraboloid is shown in the boundaries of the belt of greatest condensation. The locus of incipient rupture, Mercury's secular perihelion, is about $\frac{1}{8}$ of the locus of incipient subsidence, secular aphelion of Mars. Stockwell's values for the two loci are .2974008 and 1.736478. This gives for the major axis of the several incipient ellipses, described by the subsiding particles from the outer portion of the belt, $.2974008 + 1.736478 = 2.0338788$. Let g_0 , g_3 represent equatorial superficial gravitating acceleration of Sun, Earth, respectively; m_5 , m_3 , masses of Jupiter, Earth; t_a , time of Jupiter's orbital revolution; t_β , time of Earth's rotation; ρ_3 , Earth's semi-axis major; ρ_n , asteroidal radius equivalent to major axis of incipient ellipses of dense belt. Then

$$\frac{t_a}{t_\beta} \times \frac{m_3}{m_5} \times \frac{\rho_n}{\rho_3} = \frac{g_0}{g_3}$$

$$4332.58482 \div 316.617 \times 2.0338788 = \frac{g_0}{g_3} = 27.8316$$

$$\frac{m_0}{m_3} \times \frac{g_3}{g_0} = \left(\frac{r_0}{r_3} \right)^2$$

$$331776 \div 27.8316 = 11920.8$$

$$\frac{r_0}{r_3} = \sqrt{11920.8} = 109.183$$

$$r_0 = 109.183 \times 3962.8 = 432669 \text{ miles.}$$

$$\rho_3 \div r_0 = 92785700 \div 432669 = 214.45$$

These results may be compared with those which were given in Notes 91, 113 and 156, the extreme range of difference being less than $\frac{1}{10}$ of one per cent.

169. *Earth's Incipient Subsidence.*

If the various relations which are shown in the foregoing note are due to Earth's atmospheric and nucleal subsidence from the centre of the dense belt $\left(\frac{\rho_n}{2} = 1.0169394\right)$, its secular aphelion should be $\left(\frac{\rho_n}{2}\right)^4 = 1.0695$. Stockwell gives (*op cit.*, p. 38) 1.0677352, upon the assumption that $\frac{m_0}{m_3} = 368689$. On page xi of his Introduction he gives 1.0693888; on page xvii he gives a series of values which yield, by interpolation, 1.0691 for the photodynamic mass-ratio, $\frac{m_0}{m_3} = 331776$.

170. *Progression of Fundamental Atomicities.*

Thomas Bailey, (*Phil. Mag.*, Jan. 1882, p. 35), gives a series of atomic weights corresponding to minimum volumes, which are members of the geometric series a, ab, ab^2, ab^3, ab^4 , the value of b being $\frac{1}{6}$ of a and the value of a being 10. This suggests an atomic parabolic motion, like that in the photodynamic or interstellar paraboloid, in which $\xi = \frac{1}{6}$. We may also notice that 6 is the product of the two phyllotactic numbers, 2 and 3.

171. *Perissad Phyllotaxy.*

The indications of phyllotactic tendency in various departments of physics, have induced me to test Gerber's groupings of chemical atoms by methods which seem to me to be perfectly legitimate. In order to remove all effects of personal equation or bias, as well as of accidental or empirical coincidence, I adopt Clarke's recalculation of atomic weights (*Phil. Mag.* [5] 12, 109-10), and my strictly phyllotactic divisors (Note 136), instead of Gerber's empirical divisors. In view of the *a priori* probability of tendency to division in extreme and mean ratio, I assume that the ratio of probability to improbability, in each instance, is equivalent to at least $\frac{1}{4} D : (T-O)$; D being the phyllotactic divisor, T the theoretical atomic weight or nearest exact multiple of D , and O the observed atomic weight taken from Clarke's table. I have added Rb and Tl to Gerber's list of monatomic elements, and Bo to his trivalent list.

Monatomic Group ; $D_1 = .768$.

		T.	O.	T-O.	Probability.
Li	9 D_1	6.912	7.007	.095	192 : 95
Na	30 D_1	23.040	22.998	.042	192 : 42
K	51 D_1	39.168	39.019	.149	192 : 149
Cs	173 D_1	132.864	132.583	.281	192 : 281
Fl	25 D_1	19.200	18.984	.216	192 : 216
Cl	46 D_1	35.328	35.370	.042	192 : 42
Br	104 D_1	79.872	79.768	.104	192 : 104
I	165 D_1	126.720	126.557	.163	192 : 163
Ag	140 D_1	107.520	107.675	.155	192 : 155
Rb	111 D_1	85.248	85.251	.003	192 : 3
Tl	265 D_1	203.520	203.715	.195	192 : 195

Three of the elements, Cs, Fl and Tl, indicate a probability that the phyllotactic approximation may be merely accidental. The aggregate probability that the combining equivalents of the monatomic elements are modified by phyllotactic tendencies, or the product of all the separate probabilities, is more than 5610 times as great as the probability that the approximations are accidental.

Trivalent Group ; $D_3 = 1.559$.

		T.	O.	T-O.	Probability.
N	9 D_3	14.031	14.021	.010	389.75 : 10
P	20 D_3	31.180	30.958	.222	389.75 : 222
As	48 D_3	74.832	74.918	.086	389.75 : 86
Sb	77 D_3	120.043	119.955	.088	389.75 : 88
Bi	133 D_3	207.347	207.523	.176	389.75 : 176
Au	126 D_3	196.434	196.155	.279	389.75 : 279
Bo	7 D_3	10.913	10.941	.028	389.75 : 28

All the indications in this group are in favor of phyllotactic influences, the aggregate ratio of probabilities being more than 108426 : 1. Multiplying this by the monatomic ratio we get, for the aggregate perissad ratio, > 668375000 : 1.

*173. Artiad Phyllotaxy.**Di- or Tetratomic Group ; $D_2 = 1.996$.*

		T.	O.	T-O.	Probability.
O	8 D_2	15.968	15.963	.005	499 : 5
S	16 D_2	31.936	31.984	.048	499 : 48
Se	40 D_2	79.840	78.797	1.043	499 : 1043
Te	64 D_2	127.744	127.960	.216	499 : 216
Mg	12 D_2	23.952	23.959	.007	499 : 7
Ca	20 D_2	39.920	39.990	.070	499 : 70
Sr	44 D_2	87.824	87.374	.450	499 : 450
Ba	69 D_2	137.724	136.763	.961	499 : 961
C	6 D_2	11.976	11.974	.002	499 : 2

		T.	O.	T-O.	Probability.
Si	14 D ₂	27.944	28.195	.251	499 : 251
Ti	25 D ₂	49.900	49.846	.054	499 : 54
Zr	45 D ₂	89.820	89.367	.453	499 : 453
Sn	59 D ₂	117.764	117.698	.066	499 : 66
Hg	100 D ₂	199.600	199.712	.112	499 : 112
Mo	48 D ₂	95.808	95.527	.281	499 : 281
W	92 D ₂	183.632	183.610	.022	499 : 22
U	60 D ₂	119.760	119.241	.519	499 : 519

Two of these elements, Se and U, give adverse indications ; the aggregate ratio of favorable to adverse probabilities is more than 17173770000000 : 1. I have taken $\frac{1}{2}$ of Clarke's estimate for U, in order to compare it with Gerber's assumed atomicity.

Supplementary Artiad Group.

Barker, in *Johnson's Cyclopædia*, gives other artiad elements which Gerber places in his group of metals. In order to complete the comparisons which are based upon valency they are inserted here :

		T.	O.	T-O.	Probability.
Gl	7 D ₂	13.972	13.695	.277	499 : 277
Al	14 D ₂	27.944	27.009	.935	499 : 935
In	57 D ₂	113.772	113.398	.374	499 : 374
Zn	33 D ₂	65.868	64.905	.963	499 : 963
Cd	56 D ₂	111.776	111.770	.006	499 : 6
Cu	32 D ₂	63.872	63.173	.699	499 : 699
Pb	103 D ₂	205.588	206.471	.883	499 : 883
Pd	53 D ₂	105.788	105.737	.051	499 : 51
Pt	97 D ₂	193.612	194.415	.803	499 : 803
Yt	45 D ₂	89.820	89.816	.004	499 : 4
Ce	70 D ₂	139.720	140.424	.704	499 : 704
La	69 D ₂	137.724	138.526	.802	499 : 802
Di	72 D ₂	143.712	144.573	.861	499 : 861
Er	83 D ₂	165.668	165.891	.223	499 : 223
Th	117 D ₂	233.532	233.414	.118	499 : 118
Cr	26 D ₂	51.896	52.009	.113	499 : 113
Fe	28 D ₂	55.888	55.913	.025	499 : 25
Mn	27 D ₂	53.892	53.906	.014	499 : 14
Ni	29 D ₂	57.884	57.928	.044	499 : 44
Co	30 D ₂	59.880	58.887	.993	499 : 993
Ru	52 D ₂	103.792	104.217	.425	499 : 425
Rh	52 D ₂	103.792	104.055	.263	499 : 263
Ir	97 D ₂	193.612	192.651	.961	499 : 961
Os	99 D ₂	197.604	198.494	.890	499 : 890

The first sub-group, Glucinum to Thorium, inclusive, consists of dyads

and tetrads, and gives 40911 : 1 for the combined ratio of probabilities. The other sub-group is hexad, giving the aggregate ratio 11611 : 1. The total aggregate ratio of the artiad elements is more than 81585(10)¹⁷ : 1.

173. *Metallic Phyllotaxy.* $D_4 = 1.247$.

		T.	O.	T-O.	Probability.
Gl	11 D_4	13.717	13.972	.255	311.75 : 255
Al	22 D_4	27.434	27.009	.425	311.75 : 425
Sc	35 D_4	43.645	43.980	.335	311.75 : 335
Cr	42 D_4	52.374	52.009	.365	311.75 : 365
Fe	45 D_4	56.115	55.913	.202	311.75 : 202
Ga	55 D_4	68.585	68.854	.269	311.75 : 269
In	91 D_4	113.477	113.398	.079	311.75 : 79
Zn	52 D_4	64.844	64.905	.061	311.75 : 61
Cd	90 D_4	112.230	111.770	.460	311.75 : 460
Mn	43 D_4	53.621	53.906	.285	311.75 : 285
Ni	46 D_4	57.362	57.928	.566	311.75 : 566
Co	47 D_4	58.609	58.887	.278	311.75 : 278
Cu	51 D_4	63.597	63.173	.424	311.75 : 424
Pb	166 D_4	207.002	206.471	.531	311.75 : 531
Tl	163 D_4	203.261	203.715	.454	311.75 : 454
Rb	68 D_4	84.796	85.251	.455	311.75 : 455
Ru	84 D_4	104.748	104.217	.531	311.75 : 531
Rh	83 D_4	103.501	104.055	.554	311.75 : 554
Pd	85 D_4	105.995	105.737	.258	311.75 : 258
Ir	154 D_4	192.038	192.651	.613	311.75 : 613
Pt	156 D_4	194.532	194.415	.117	311.75 : 117
Os	159 D_4	198.273	198.494	.221	311.75 : 221
Yt	72 D_4	89.784	89.816	.032	311.75 : 32
Ce	113 D_4	140.911	140.424	.487	311.75 : 487
La	111 D_4	138.417	138.526	.109	311.75 : 109
Di	116 D_4	144.652	144.573	.079	311.75 : 79
Er	133 D_4	165.851	165.891	.040	311.75 : 40
Th	187 D_4	233.189	233.414	.225	311.75 : 225

The aggregate ratio is 1386.8 : 1, the mean ratio for a single comparison being somewhat less than 4 : 3. The indication of phyllotactic tendency is, therefore, comparatively slight, and far less satisfactory than in the grouping according to valency.

174. *General Test of Atomic Phyllotaxy.*

Computers who are accustomed to calculations of probable error, and who have not given any special attention to the harmonic influences of æthereal vibrations, may, perhaps, question the propriety of making any allowance for an *a priori* probability of division in extreme and mean ratio. For the satisfaction of all doubts upon this point it may be well to apply

some test which will be rigid enough to fulfill the broadest requirements of mathematical likelihood. If we substitute $\frac{3}{8} D$, for $\frac{1}{4} D$, in the ratio of probability to improbability, we provide for requirements of linear oscillation, orbital motion and gravitating tendency. In such limited ranges of comparison as are possible for the chemical elements, most mathematicians would, perhaps, be satisfied with this substitution. All doubt should be removed by introducing the coefficient of probable error, .674489, and using $.674489 \times \frac{1}{4} D = .168622 D$. If we let n represent the number of terms in a given group, the ratios of probability, which have been found in Notes 171-3, should be multiplied by .674489ⁿ, in order to give results which are entirely independent of any *a priori* assumption. We then find

For the monatomic group, Note 171	73.75 : 1
“ trivalent “ “ 171	6885.88 : 1
“ di-ortetramic “ “ 172	21253910000.00 : 1
“ supp’y artiad “ “ 172	37337.33 : 1
“ aggregate valency	403(10) ¹⁸ : 1
“ metallic group, Note 173	1 : 44.33

The mean ratios, for single representatives of the several groups, are the following :

For the monatomic group	1.478 : 1
“ trivalent “	3.534 : 1
“ 1st artiad “	4.050 : 1
“ 2d “ “	1.550 : 1
“ aggregate valency	2.235 : 1
“ metallic group	1 : 1.145

The uniform character of the phyllotactic indications, in the groupings which are based upon similitudes of chemical affinity, is very satisfactory. To all who are willing to attach weight to *a priori* considerations, the following statement of mean ratios may be acceptable :

For the monatomic group, Note 171	2.192 : 1
“ trivalent “ “ 171	5.223 : 1
“ 1st artiad “ “ 172	6.005 : 1
“ 2d “ “ “ 172	2.299 : 1
“ aggregate valency	3.313 : 1
“ metallic group, “ 173	1.295 : 1
“ perissads	3.076 : 1
“ artiads	3.423 : 1
“ hydrogen unit	2.084 : 1

The last result was quite unexpected. It was obtained by assuming .250 as a probable mean difference from exact multiples of H, and treating all the values in Clarke’s table in the same way as in the phyllotactic examinations of Notes 171-3, so as to obtain, for each element, the ratio, $\frac{1}{4} H : (T-O)$. Although the aggregate evidence of phyllotactic influence upon valency, (3.313 : 1), is nearly 1.6 times as great as the evidence of

hydrogenic influence upon general atomicity, the mathematical probability of the latter is satisfactorily established. I am not aware that the views of Prout and Dalton have ever before been tested in any way like this.

175. *Combination of Harmonic Influences.*

In my studies of cosmical harmony I have often had occasion to speak of the simultaneous operation of different oscillatory tendencies. Similar tendencies, involving similar modifications of resulting rhythms, must exist in the various forms of molecular activity. Dr. Thomas Hill, whose participation in Peirce's investigations of planetary phyllotaxy have given him an interest in other like researches, having suggested that the surd, $\frac{1}{3}(3-\sqrt{5})$, might be more closely represented in the atomic ratios than its phyllotactic approximations, I have tried it upon each of the foregoing groups. I find some evidence of its influence, but the combinations of phyllotactic ratios which are represented by my two divisors, .768 and 1.996, are much more satisfactory. Therefore it seems probable that, although the differences of internal work may prevent any precise atomic commensurability, there are as close approximations to precision in the elementary atoms as there are in plants and in planets.

176. *Fourier's Doctrine of Elasticity.*

The early views of Rittenhouse and other American investigators,* are corroborated by the following extract from Fourier's "*Theorie analytique de la chaleur*," which is cited by Melsens in his report on Hirn's experimental investigations of the relation which exists between the resistance of the air and its temperature (*Bull. de l'Acad. Roy. de Belgique*, [3] 2, p. 252, 8 Octobre 1881).

Art. 53. "La chaleur est le principe de toute élasticité ; c'est sa force répulsive qui conserve la figure des masses solides et le volume des liquides. Dans les substances solides, les molécules voisines céderaient à leur attraction mutuelle, si son effet n'était pas détruit par la chaleur qui les sépare. Cette force élastique est d'autant plus grande que la température est plus élevée ; c'est pour cela que les corps se dilatent ou se condensent, lorsqu'on élève ou lorsqu'on abaisse leur température."

177. *Test of Atomic Divisors by Arithmetical Means.*

The superiority of the combined phyllotactic divisors, over the surd divisors, Gerber's empirical divisors and the hydrogen unit, may be further shown by comparing the mean percentages of difference from exact multiples of the several divisors, in each of Gerber's groups :

	$\frac{1}{2}(3-\sqrt{5})$.	$\frac{1}{2}(\sqrt{5}-1)$.	H.	Gerber.	Phyllo- tactic.
For the monatomic group	.2549	.2404	.2034	.2140	.1804
“ trivalent “	.2322	.2312	.1303	.0878	.0878
“ di- or tetratomic “	.2385	.2755	.2140	.1072	.1044
“ metallic “	.2598	.2543	.2342	.2635	.2546
“ combined aggregate	.2501	.2563	.2086	.1931	.1847

* See Proc. Amer. Phil. Soc., xvi, 298 *seq.*

We find, therefore, that in this comparison the evidence of phyllotactic influence upon valency is more striking than that of the hydrogen unit or of Gerber's empirical divisors. In Gerber's special metallic group, however, the hydrogen unit furnishes the nearest, and Gerber's divisor the most remote approximation.

178. *Probable Errors of Atomic Remainders.*

If the deviations from exact multiples of the several divisors are treated as errors of observation, in order to determine the "probable error," we get the following results :

	Accidental.	H.	Gerber. Phyllotactic.	
For the perissads	$\pm .0502$	$\pm .0433$	$\pm .0371$	$\pm .0300$
“ artiad	$\pm .0294$	$\pm .0280$	$\pm .0178$	$\pm .0180$
For all the elements	$\pm .0266$	$\pm .0236$	$\pm .0163$	$\pm .0154$

The legitimacy of this treatment may be questioned, but it cannot be charged with any unjust partiality. The artiad furnish an instance in which Gerber's empirical divisors give the nearest approximation. The hydrogen unit is still the least satisfactory of all.

179. *Deduced Laws of Atomicity.*

Notes 171-8 seem to justify the following conclusions :

1. If all the atomic weights were accurately determined, they would be found to be exact multiples of the hydrogen unit.
2. Chemical combinations are influenced by phyllotactic laws, or by tendencies to division in extreme and mean ratio.
3. Artiad and perissad combining units are different, but connected by phyllotactic ratios.
4. Metallic structure is controlled by phyllotactic laws.

180. *Phyllotactic Relations to Oxygen.*

In Note 138 I showed that 37 of the elements, according to Clarke's table, may be more nearly measured by $\frac{1}{18}$ O, while 26 approximate more closely to exact multiples of H. If we take $\frac{1}{8}$ O = 1.995 H, we get Gerber's di- or tetratomic divisor, from which others may be deduced by simple phyllotactic ratios :

Phyllotactic.		Gerber.
α	1.995	D_2 1.995
$\beta = \frac{1}{2} \alpha$.9975	H .9997
$\gamma = \frac{5}{13} \alpha$.7673	D_1 .769
$\delta = \frac{8}{9} \alpha$	1.2469	D_4 1.245
$\epsilon = \frac{3}{4} \delta$	1.5586	D_3 1.559

The following comparative tables introduce all of Clarke's recalculated atomic weights :

Monatomic ; β , γ , H, D_1 .

Phyllotactic.			Gerber.		Clarke.	Δ_1	Δ_2	Δ_3
H	β	.9975	H	.9997	1.0000	.0000	.0025	.0003
Li	9 γ	6.9357	9 D_1	6.921	7.0073	.0010	.0147	.0125
Na	30 γ	23.0190	30 D_1	23.070	22.998	.0001	.0009	.0031
K	51 γ	39.1323	51 D_1	39.219	39.019	.0005	.0029	.0051
Cs	173 γ	132.7429	172 D_1	132.268	132.583	.0031	.0012	.0024
Fl	25 γ	19.1825	25 D_1	19.225	18.984	.0008	.0103	.0125
Cl	46 γ	35.2958	46 D_1	35.374	35.370	.0106	.0021	.0001
Br	104 γ	79.7992	104 D_1	79.976	79.768	.0029	.0004	.0026
I	165 γ	126.6045	165 D_1	126.885	126.557	.0035	.0004	.0026
Ag	140 γ	107.4220	140 D_1	107.660	107.675	.0030	.0024	.0001
Tl	265 γ	203.3345	265 D_1	203.785	203.715	.0014	.0019	.0003
Rb	111 γ	85.1703	111 D_1	85.359	85.251	.0029	.0009	.0013

I have added Tl and Rb to the elements which Gerber included in his monatomic group. Δ_1 , Δ_2 , Δ_3 , are the ratios of the differences between H, γ and D_1 , respectively, and the values which may be found by a division of Clarke's atomicities by the theoretical atomicities. The arithmetical mean values correspond with the order of arrangement, viz. : Δ_1 , .0028 ; Δ_2 , .0034 ; Δ_3 , .0036.

Di- or Tetratomic ; $a = D_2$.

Phyl., Gerber.			Clarke.	Δ_1	$\Delta_2 = \Delta_3$
O	8 D_2	15.96	15.9633	.0023	.0002
S	16 D_2	31.92	31.984	.0005	.0020
Se	39 D_2	77.805	78.797	.0026	.0126
Te	64 D_2	127.68	127.960	.0003	.0022
Mg	12 D_2	23.94	23.959	.0017	.0008
Ca	20 D_2	39.90	39.990	.0002	.0023
Sr	44 D_2	87.78	87.374	.0043	.0046
Ba	69 D_2	137.655	136.763	.0017	.0065
C	6 D_2	11.97	11.9736	.0022	.0003
Si	14 D_2	27.93	28.195	.0069	.0095
Ti	25 D_2	49.875	49.846	.0031	.0006
Zr	45 D_2	89.775	89.367	.0941	.0046
Sn	59 D_2	117.705	117.698	.0026	.0001
Hg	100 D_2	199.50	199.712	.0014	.0011
Mo	48 D_2	95.76	95.527	.0049	.0024
W	92 D_2	183.54	183.610	.0021	.0004
U	120 D_2	239.40	238.482	.0020	.0038

These are the same elements as are embraced in Gerber's second group. The arithmetical mean values of the deviations are, Δ_1 , .0025 ; Δ_2 , Δ_3 , .0033.

Tri- or Pentavalent ; ε , D_3 .

	Phyllotactic.		Gerber.		Clarke.	Δ_1	Δ_2	Δ_3
N	9 ε	14.0274	9 D_3	14.031	14.021	.0015	.0005	.0007
P	20 ε	31.1720	20 D_3	31.180	30.958	.0014	.0069	.0071
As	48 ε	74.8128	48 D_3	74.832	74.918	.0011	.0014	.0012
Sb	77 ε	120.0122	77 D_3	120.043	119.955	.0004	.0005	.0007
Bi	133 ε	207.2938	133 D_3	207.347	207.523	.0023	.0011	.0009
Au	126 ε	196.3836	126 D_3	196.434	196.155	.0008	.0012	.0014
Bo	7 ε	10.9102	7 D_3	10.913	10.941	.0054	.0028	.0026
Ta	117 ε	182.3562	117 D_3	182.403	182.144	.0008	.0012	.0014
V	33 ε	51.4338	33 D_3	51.447	51.256	.0050	.0035	.0037

I have added Bo, Ta and V to the elements which Gerber included in this group. The arithmetical mean values of the deviations are, Δ_1 , .00208 ; Δ_2 , .00212 ; Δ_3 , .00215.

Metallic ; δ , D_4 .

	Phyllotactic.		Gerber.		Clarke.	Δ_1	Δ_2	Δ_3
G1	7 δ	8.7283	7 D_4	8.715	9.085	.0094	.0408	.0411
Al	22 δ	27.4318	22 D_4	27.390	27.009	.0003	.0154	.0141
Sc	35 δ	43.6415	35 D_4	43.575	43.980	.0005	.0078	.0092
Cr	42 δ	52.3698	42 D_4	52.290	52.009	.0002	.0069	.0054
Fe	45 δ	56.1105	45 D_4	56.025	55.913	.0016	.0035	.0020
Ga	55 δ	68.5795	55 D_4	68.475	68.854	.0021	.0040	.0055
In	91 δ	113.4679	91 D_4	113.295	113.398	.0035	.0006	.0009
Zn	52 δ	64.8388	52 D_4	64.740	64.905	.0015	.0010	.0025
Cd	90 δ	112.2210	90 D_4	112.050	111.770	.0021	.0040	.0025
Mn	43 δ	53.6167	43 D_4	53.535	53.906	.0017	.0054	.0069
Ni	46 δ	57.3574	47 D_4	58.515	57.928	.0012	.0099	.0101
Co	47 δ	58.6043	47 D_4	58.515	58.887	.0019	.0048	.0063
Cu	51 δ	63.5919*	51 D_4	63.495	63.173	.0027	.0066	.0051
Pb	166 δ	206.9854	166 D_4	206.670	206.471	.0023	.0025	.0010
Ru	84 δ	104.7396	84 D_4	104.580	104.217	.0021	.0050	.0035
Rd	83 δ	103.4927	84 D_4	104.580	104.055	.0005	.0034	.0050
Pd	85 δ	105.9865	85 D_4	105.825	105.737	.0025	.0024	.0008
Ir	155 δ	193.2695	155 D_4	192.975	192.651	.0018	.0032	.0017
Pt	156 δ	194.5164	156 D_4	194.220	194.415	.0021	.0005	.0010
Os	159 δ	198.2571	159 D_4	197.955	198.494	.0025	.0012	.0027
Yt	72 δ	89.7768	72 D_4	89.640	89.816	.0020	.0004	.0020
Ce	113 δ	140.8997	113 D_4	140.685	140.424	.0030	.0034	.0019
La	111 δ	138.4059	111 D_4	138.195	138.526	.0034	.0009	.0024
Di	116 δ	144.6404	116 D_4	144.420	144.573	.0029	.0005	.0011
Er	133 δ	165.8377	133 D_4	165.585	165.891	.0007	.0003	.0018
Th	187 δ	233.1703	187 D_4	232.815	233.414	.0018	.0010	.0026
Ytter	139 δ	173.3191	139 D_4	173.055	172.761	.0014	.0032	.0017

The mean deviations are, Δ_1 , .00227 ; Δ_2 , .00521 ; Δ_3 , .00515. In order

to complete the data for comparisons of probable error, I repeat this group, with artiad divisors.

Metallic, II; $\alpha = D_2$.

G1	5 D_2	9.975	.0890	Ru	52 D_2	103.740	.0046
Al	14 D_2	27.930	.0329	Rd	52 D_2	103.740	.0030
Sc	22 D_2	43.890	.0020	Pd	53 D_2	105.735	.0000
Cr	26 D_2	51.870	.0027	Ir	97 D_2	193.515	.0045
Fe	28 D_2	55.860	.0009	Pt	97 D_2	193.515	.0046
Ga	35 D_2	69.825	.0139	Os	99 D_2	197.505	.0050
In	57 D_2	113.715	.0028	Yt	45 D_2	89.775	.0005
Zn	33 D_2	65.835	1.041	Ce	70 D_2	139.650	.0055
Cd	56 D_2	111.720	.0004	La	69 D_2	137.655	.0063
Mn	27 D_2	53.865	.0008	Di	72 D_2	143.640	.0065
Ni	29 D_2	57.855	.0013	Er	83 D_2	165.585	.0018
Co	30 D_2	59.850	.0161	Th	117 D_2	233.415	.0000
Cu	32 D_2	63.840	.0104	Ytter	87 D_2	173.565	.0046
Pb	103 D_2	205.485	.0048				

The mean deviation is .00885.

181. *Comparative Summary.*

Although I have shown in Note 149, that Schuster's test will often fail to detect harmonies which really exist, it may be used with advantage in many instances of comparative probability. The following tables seem to furnish indisputable evidence of phyllotactic influence upon atomicity.

Logarithms of Probability.

Groups.	Phyllotactic.	Gerber.	Hydrogen.
Monatomic,	3.7476668	3.6572183	5.1634821
Tri- and Pentavalent,	5.2591327	5.2591327	3.8626605
Di- and Tetratomic,	12.0650575	11.9044217	5.1740365
Metallic,	3.0609802	1.2331805	6.1850405
Aggregate,	24.1328372	22.0539532	20.3852196
Mean,	.3770756	.3445930	.3185191

Arithmetical Residuals.

Groups.	Phyllotactic.	Gerber.	Hydrogen.
Monatomic,	.1672	.1947	.2842
3 and 5,	.0926	.0926	.1423
2 and 4,	.1059	.1442	.2119
Metallic,	.2487	.2523	.2247
Mean,	.1748	.1912	.2199

Relative Probability.

Groups.	Phyllotactic.	Gerber.	Hydrogen.
Monatomic,	1.2315	1.0000	32.0822
3 and 5,	24.9156	24.9156	1.0000
2 and 4,	7780740.	5375080.	1.0000
Metallic,	67.2666	1.0000	89507.6
Aggregate,	5592.649	46.637	1.0000
Mean,	1.144	1.062	1.0000

Relative Residuals,

Groups.	Phyllotactic.	Gerber.	Hydrogen.
Monatomic,	1.0000	1.1645	1.6998
3 and 5,	1.0000	1.0000	1.5367
2 and 4,	1.0000	1.3617	2.0009
Metallic,	1.1068	1.1229	1.0000
Mean,	1.0000	1.0938	1.2580

The "logarithms of probability" are deduced from the first four groupings of Note 180, by the method and with the phyllotactic values which were adopted in Notes 171-4. They assign a greater degree of importance to the strictly phyllotactic than to Gerber's approximately phyllotactic divisors, in every instance; a greater degree of importance to the hydrogen divisor than to either the strictly phyllotactic or the approximately phyllotactic divisors, in the monatomic and metallic groups; a greater degree of importance both to the phyllotactic and to Gerber's divisors than to the hydrogen divisor, in the 3 and 5, 2 and 4, aggregate and mean groups.

The "arithmetical residuals" are deduced from the first four groupings of Note 180, by dividing the differences from exact multiples of the several divisors by the respective divisors, by the method which was adopted in Note 177. In this aspect of the question, as in Note 177, the hydrogen unit is most important in the metallic group; the phyllotactic divisors, in each of the other groups; Gerber's coinciding with the phyllotactic in the tri- and pentavalent groups.

The "relative probability" and "relative residuals" are found by taking the least value in each group as the unit. The indications are, of course, the same as in the systems of grouping from which they were derived. In the metallic group the probability of predominant hydrogen influence is 89507.6 times as great as that of Gerber's divisors, or 1330.6 times as great as that of the phyllotactic divisors. In the di- and tetratomic group the phyllotactic probability is more than 7780740 times as great as that of the hydrogen divisor, or 1.4475 times as great as that of Gerber's divisors. The aggregate phyllotactic probability is 15592.649 times as great as that of the hydrogen divisor, or 119.918 times as great as that of Gerber's divisors.

182. Synopsis of Probable Errors.

The following tables are computed on the hypothesis that the atomic weights are exact multiples of the several divisors. The percentages of the divisors which represent $(T - O) \div D$,* are treated as errors of observation, and the probable errors are deduced in the usual way. Those percentages may evidently vary between 0 and $\pm .5$.

* See Note 171.

Probable Errors.

Groups.	Surd I.	Surd II.	Hydrogen.	Gerber.	Phyllotactic.
Monatomic,	$\pm .0529$	$\pm .0545$	$\pm .0552$	$\pm .0490$	$\pm .0407$
3 and 5,	$\pm .0588$	$\pm .0556$	$\pm .0445$	$\pm .0253$	$\pm .0253$
2 and 4,	$\pm .0438$	$\pm .0497$	$\pm .0439$	$\pm .0357$	$\pm .0218$
Metallic,	$\pm .0375$	$\pm .0385$	$\pm .0358$	$\pm .0358$	$\pm .0361$
Aggregate,	$\pm .0232$	$\pm .0242$	$\pm .0222$	$\pm .0200$	$\pm .0181$

Relative Probable Errors.

Groups.	Surd I.	Surd II.	Hydrogen.	Gerber.	Phyllotactic.
Monatomic,	1.3001	1.3401	1.3571	1.2038	1.0000
3 and 5,	2.3211	2.1933	1.7559	1.0000	1.0000
2 and 4,	2.0101	2.2764	2.0109	1.6361	1.0000
Metallic,	1.6473	1.0752	1.0004	1.0000	1.0085
Aggregate,	1.2812	1.3394	1.2274	1.1079	1.0000

Surd I is $\frac{1}{2}(3 - \sqrt{5})$; Surd II, $\frac{1}{2}(\sqrt{5} - 1)$. The groupings and the phyllotactic divisors are the same as in the foregoing note. Phyllotactic precedence is shown in eight of the groups; Gerber's approximately phyllotactic in four; hydrogen in one. Surd divisors take precedence of hydrogen in four of the groups. They suggest the probability that dextro- and lævo-gyration may be phyllotactic phenomena, originating in tendencies to division in extreme and mean ratio.

183. *Probabilities of the Surd Divisors.*

In order to show the character of the evidence to which I referred in Note 175, and thus complete the comparative examination which I have undertaken, I add the following tables.

Logarithms of Probability. Arithmetical Residuals.

Groups.	Surd I.	Surd II.	Surd I.	Surd II.
Monatomic,	.6805664	.9997215	.240	.239
3 and 5,	1.4766962	1.1496250	.231	.226
2 and 4,	3.4829337	.1602944	.238	.275
Metallic,	1.7491278	1.4979853	.260	.266
Aggregate,	7.3893241	3.8076262	.247	.258
Mean,	.1154582	.0594942

The arithmetical residuals are so near Schuster's limit that they furnish but slight evidence of harmonic influence. The mean probabilities, 1.304 and 1.147, are also comparatively small, but inasmuch as each of the groups indicates a decided probability, while the aggregates are more than 24,500,000 and 6420 respectively, any hypothesis of accidental determination must be rejected. Perhaps the most important use which we can make of the results is to extend the comparisons of the foregoing note, so as to add further cogency to the proof of phyllotactic sway.

Relative Probability.

	Phyllotactic.	Gerber.	Hydrogen.	Surd I.	Surd II.
Aggregate	21145(10) ¹⁶	1763(10) ¹⁵	378(10) ¹⁴	3816.8	1
Mean	2.078	1.928	1.815	1.138	1
Residual	1.476	1.349	1.173	1.044	1

184. Another Comparative Summary.

If we take the percentages of deviation, instead of the fractional deviations from exact multiples of the several divisors, and divide by the number of hydrogen units which most nearly represents each of Clarke's atomic weights, we obtain data for computing other probabilities and probable errors, which are given below :

Logarithms of Relative Probability.

Groups.	Phyllotactic.	Gerber.	Hydrogen.	Surd I.	Surd II.
Monatomic	5.0490205	4.6466652	5.7455887	.0000000	2.7689906
3 and 5	4.5555159	4.5555159	2.8622287	.0000000	.0090219
2 and 4	12.2941388	10.7902815	4.5435830	1.0009281	.0000000
Metallic	.5855468	.0000000	4.1708538	.2005275	.2278172
Aggregate	21.2824664	18.7910070	16.1207986	.0000000	1.8043741
Mean	.3325385	.2936095	.2518875	.0000000	.0281933

Probable Errors.

Groups.	Phyllotactic.	Gerber.	Hydrogen.	Surd I.	Surd II.
Monatomic	± .00117	± .00153	± .00100	± .00393	± .00328
3 and 5	± .00042	± .00042	± .00059	± .00290	± .00264
2 and 4	± .00035	± .00039	± .00050	± .00221	± .00161
Metallic	± .00065	± .00098	± .00117	± .00076	± .00096
Aggregate	± .00036	± .00051	± .00028	± .00103	± .00090

185. Incipient Phyllotaxy.

The probable errors both in Note 182 and in Note 184, seem to give more indications than are furnished by the relative probabilities, of surd influence upon atomicity. There is room, however, for a reasonable doubt whether those indications are other than accidental, and it would undoubtedly be desirable, if it were possible, to find some more satisfactory test of probabilities which are so near to the boundary line between normal and casual coincidences. In the aggregate probabilities, hydrogen stands between the surd divisors and the phyllotactic divisors. The latter were tested by elements which are denser than hydrogen, and, therefore, have a greater atomic inertia. Is it not likely that the former may find their rightful province in a more æthereal region, either in the primitive "subsidence" of nebulous matter or in the undulations which precede subsidence? Cyclical tendencies may naturally become more marked as solidification increases.

186. *Foreshadowings.*

The greatest superiority of S_1 over S_2 , as well as of the phyllotactic and approximately phyllotactic divisors over the hydrogen divisor, is found in the di- and tetratomic group. The aggregate probability of hydrogen influence on atomicity is more than 9,900,000,000,000 times as great as that of S_1 , or more than 37,800,000,000,000,000 times as great as that of S_2 , or more than 242,700,000,000,000,000,000 times the probability of accidental coincidence. Each of these numbers should be multiplied by 5592.6 to give the probability of the phyllotactic divisors. The lowest surd probability in either group is that of S_2 in the di- and tetratomic group, which is only 1.446 : 1, or a little more than 13 : 9. Even this ratio, however, is satisfactory as indicative of incipient action, and suggestive of researches in the "nascent state," the "fourth state of matter," or in some other approximation to the æthereal condition. The artiad and metallic elements seem also to offer fields for important future discovery in regard to modifications of phyllotactic tendency by condensation or combination.

187. *Hydrogen Shares the Phyllotactic Probabilities.*

In the foregoing notes I have treated hydrogen as outside the phyllotactic group, in order to find the probability of the hypotheses of Dalton and Prout as compared with other reasonable hypotheses. As a member of the phyllotactic group, and the most general of the phyllotactic divisors, it shares all the probabilities of the group. Therefore, if there is any value in mathematical tests, the views of Berzelius, Turner, Marignac, Stas and Clarke, as to the importance of the hydrogen atom, should be accepted, rather than those of Thomas Thomson and Dumas.

188. *Inertia and Elasticity.*

Thus the evidences are multiplying, in every direction, of the importance of giving great heed to the blended sway of inertia and elasticity, in all physical researches. The moment of inertia is of especial importance, inasmuch as material particles, in an elastic medium, become the seats of living forces which enable us to apply the laws of composition and resolution of forces, to composition and resolution of motions. The principles which I applied successfully, in 1863, to barometric estimates of the Sun's mass and distance, have been abundantly exemplified in every field in which I have sought for evidences of their influence, and now they are found at the very threshold of material structure, where cohesive and chemical attractions first show themselves.

189. *Phyllotaxy of Central Force.*

I have already spoken of the appearance of the phyllotactic numbers 1, 2, 3, 5 and 8, in crystallization, and especially in the mimicry of frost-pictures. The simplest phenomena of central force introduce the first three phyllotactic powers; Kepler's third law gives the phyllotactic frac-

tional exponent $\frac{3}{2}$; the actions and reactions of elasticity and inertia in nucleation, assign, as I have shown, the product of two phyllotactic exponents, $2 \times \frac{3}{2} = 3$, in the ratio of variability between the nucleal radius and Laplace's limiting radius; the relations between density and distance, in elastic media, change exponential to numerical coefficients. All of these phyllotactic relations spring from simple and elementary mathematical principles, to which the actions of central force must yield. If we call the fundamental force radiodynamic, we may be continually reminded of its alternating centripetal and centrifugal tendencies. If we call it photodynamic, the term will be naturally suggestive of the all-pervading elasticity or quasi-elasticity which propagates the rays of light and thus becomes the medium through which we get all our knowledge of heavenly bodies, as well as the largest portion of our knowledge of all earthly phenomena.

190. *Phyllotaxy of Virtual Areas.*

The planetary virtual areas, Notes 159 and 164, are jointly related through the last two of the elementary phyllotactic principles of the foregoing note. Beginning with the largest and primitively central planetary mass, the laws of nucleation and elasticity, acting first outwardly from the Sun and then inwardly, help to determine the reactionary *vis viva* of Saturn, Neptune, Uranus, Earth, Venus, Mars and Mercury, in regular succession. In passing from the extra-asteroidal to the intra-asteroidal group, another phyllotactic succession of phyllotactic ratios shows itself, the ratio between the harmonic areas* of Uranus and Earth being, within less than $1\frac{1}{4}$ per cent., $(\frac{2}{3})^6$, the exponent being the phyllotactic product 2×3 . This is also the coefficient of orbital retardation at the centre of the belt of greatest condensation; it is the $\frac{3}{4}$ power of the ratio between the harmonic areas of Jupiter and Earth, thus pointing to Uranus as a nucleal locus for which Jupiter represents Laplace's limit; the locus of secular perihelion, or incipient rupture, for Uranus, is nucleally central between the mean loci of Jupiter and Neptune; Uranus and Earth are at opposite extremities of a major-axis which would be traversed by light in the same time that Sun would rotate, if it were condensed until its present equatorial radius became Laplace's limiting radius. These five accordances present a chain of phyllotactic and photodynamic influences which seems worthy of further study.

191. *Optical and Thermal Relations in Organic Liquids.*

Brühl (*Ber. Berl. Chem. Ges.* xiv, 2533, Nov. 1881; cited in *Am. Jour. Sci.* [3], xxiii, 234), finds that progressive oxidation has the same influence on the optical as on the thermal properties of organic liquids, the refractive power diminishing as the amount of oxygen is increased, precisely as the heat of combination diminishes. Removal of hydrogen or its replacement by oxygen produces the same effect, so that both the above physical values

*The ratio of the actual virtual areas is within $\frac{1}{2\frac{1}{3}}$ of one per cent. of $(\frac{2}{3})^6$.

are greater for the hydrocarbons than for the alcohols, aldehydes, acids, etc., derived from them.

192. *Subsidiary Phyllotaxy.*

Gerber (*Les Mondes*, [3] i, 145), after referring to the accuracy with which the atomic weights of nearly one-half of the chemical elements have been determined, says :—"Un pareil degré de rigueur ne saurait être atteint dans l'application des lois d'Avogadro, de Dulong et Petit, de Mitscherlich. Celles-ci, comme il a été dit, sont des *lois de conditions*, dont nous ne possédons qu'une formule provisoire." The phyllotactic approximations are so much closer than those which are here spoken of, that we may well hope for some important results from their subsidiary employment in stoichiometry.

193. *Glucinum.*

Gerber (*l. c.* pp. 146-9), thinks that the law of Dulong and Petit accords better with the atomic weight which Nilson and Petersson assign to Glucinum, 13.65, than with the one which is adopted by Mayer and Mendelejeff, 9.1. The same thing may be said of the phyllotactic and the approximately phyllotactic divisors, for $13.65 = 11 \times 1.245 - .045 = 11 \times 1.247 - .067$, while $9.1 = 7 \times 1.245 + .385 = 7 \times 1.247 + .371$, the residuals being, respectively, 8.5 and 5.5 times as great in the latter case as in the former. This single change would increase the superiority in relative probability, of the phyllotactic divisors over the hydrogen divisor, more than twelve fold.

194. "*The Principles of Magnetism.*"

Charles Morris (*Jour. of Sci.*, [3] iv, 71) objects to the magnetic theories of Ampère and Weber, as follows : "The Ampèrian theory is constantly and gravely repeated in text-books, to the present day, without a hint being given of the indisputable fact that it is quite at variance with the principles of energy, as now understood. It is easy to imagine a constant current of electricity, and make it answer a definite purpose, but the truth is that no such thing exists as a constant current of electricity, in the Ampèrian sense." He goes on to speak of the currents of static electricity as being instantaneous, while those of galvanic and thermo-electricity consist of instantaneous components and cease when the chemical or thermal equilibrium is restored. But is the equilibrium in the terrestrial thermal and gravitating currents ever restored? In 1864 (*Proc. A. P. S.*, ix, 357, foot-note) I showed that the opposing forces of rotation, elasticity and gravitation must produce oscillations. In various preceding and subsequent papers I showed that those oscillations must produce constant currents of such descriptions as Ampère supposed.

195. *Dogmatism.*

Many modern investigators, who pride themselves on their freedom from the dreams of metaphysics, continually fall into ways which they

theoretically condemn. There is fully as much dogmatism in physics as in metaphysics. Whenever it springs from a well-grounded conviction, which has been once thoroughly tested and which always courts a repetition of tests, it is not only unobjectionable but it is highly commendable. On the other hand when it is merely theoretical, or the outgrowth of inveterate prejudice, it has no rightful place in any discussion which claims to be scientific.

196. *Numerical Tests.*

There has been an immense amount of valuable mathematical analysis which has been misunderstood, or but partially understood, for want of being properly tested. Results are never valid except for the data which they embody; they are always subject to modification by neglected, unknown or new data. The "opprobrium of thermodynamics" amounts to nothing more than the statement that, from the data which have been discussed hitherto, there appears to be a universal tendency to physical stagnation and death. The principle of equal action and reaction ought to furnish some means of escape from this opprobrium. The way of escape seems to have been indicated by the identification of a common operative velocity in light, electricity, chemistry and gravitation. A single theoretical result which has been quantitatively verified, is worth more than a thousand that are thought to be beyond the reach of verification. The theory of dependent connection between stellar rotary oscillations and the reaction of cosmical inertia against æthereal influence (Note 162 *et al.*), having been verified by the test of our Sun, seems likely to open the way for a general recognition of an æthereal reaction which will yield an exact compensation for all physical actions, affording a more satisfactory explanation of stellar light and heat than can be drawn from meteoric or shrinkage hypotheses.

197. *Velocity of Gravitating Action.*

Objections have been urged against the possibility of making gravitation the effect of light undulations unless we first overthrow Laplace's conclusion, that gravity must act with at least a hundred million times the velocity of light and that its action may be regarded as instantaneous. I answer:—
 1. I have never claimed that any physical phenomenon is the effect of another physical phenomenon, but merely that the phenomena of light and gravitation are so related as to show that they may be effects of a common cause. 2. The rapidity of action and the rapidity with which the results of the action are propagated are two entirely different things. 3. If the results of gravitating and luminous actions and reactions are identified in stellar rotations, it is altogether likely that the forces upon which these results depend act with equal speed. 4. Even if it should be found necessary to propagate gravitating undulations with a hundred million times the velocity of light, it would be as easy to suppose a gravitating æther, with a ratio of elasticity to density which is $(100,000,000)^2$ times that of the

luminiferous æther, as it is to suppose a luminiferous æther. 5. All nebular, æthereal, and other unverifiable hypotheses are useful only so far as they serve to coördinate categories of phenomena which occur as they would if the hypotheses were true. 6. I showed, long ago, that no merely physical theory or hypothesis has ever been framed which would explain the instantaneous transmission of velocity, and that such transmission, if it exists and is not physical, must be regarded as spiritual.

198. *Varying Gravitating Velocities.*

The foregoing objections may be further obviated by a consideration of the fact that gravitating velocities begin with mere tendencies to motion, and that some time must elapse before the velocity becomes appreciable. The difficulty which Faraday found, in reconciling the conservation of energy and the correlations of force with gravitating tendencies which vary inversely as the square of the distance, is a mathematical difficulty which is equally involved in heat, light, electricity and all other manifestations of radiant energy. The element of constancy may be found in a uniform elementary velocity, as in the general expression for stellar gravitating acceleration, $g_n = \frac{v_\lambda}{t_n}$ in which g_n is the acceleration of a particle at the distance n , v_λ is the velocity of light, and t_n is the time of a single oscillation or half-rotation if the star were uniformly expanded until it had a radius equal to n .

199. *Commensurability and Incommensurability.*

In his original paper (*Math. Monthly*, i, 245), Chauncey Wright said: "But if now we seek a uniform and symmetrical distribution as well as a thorough one, the interval between the successive points must be constant, and if the circumference is to be indefinitely subdivided, this interval is, of course, incommensurate." Such indefinite subdivision can hardly be looked for in any of the ordinary concrete physical phenomena, hence we find that the chemical and other approximations which we have examined are better represented by exact phyllotactic ratios than by the surd distributive tendencies. Still it seems likely that the want of precise commensurability, which is found in Clarke's table, may arise from a residual tendency to indefinite subdivision, and for this reason we may find that no increased accuracy in the determination of atomic weights will lead to the establishment of any series of divisors which are absolutely exact. I can think of no case in which the incommensurability of the surd divisors seems likely to be more completely represented than in the Ampèrian currents (Note 194).

200. *"Celestial Chemistry."*

Dr. T. Sterry Hunt (*Proc. Camb. Phil. Soc.*, reprinted in *Am. Jour. Sci.*, Feb. 1882), recites "certain views enunciated almost simultaneously by the late Sir Benjamin Brodie, of Oxford, and" himself, in the line of

development and extension of "the remarkable perception of great chemical truths which is apparent in the queries appended to the third book of Newton's *Optics*, as well as in his hypothesis touching Light and Color." Brodie's first announcement of the assumed existence of certain ideal elements was read before the Royal Society, May 3, 1866, and in the Spring of 1867 Hunt "spent several days in Paris with the late Henri Sainte-Claire Deville, repeating with him some of his remarkable experiments in chemical dissociation, the theory of which [they] then discussed in its relations to Faye's solar hypothesis." I first invited attention to the "nascent" cosmical equation, or the equation which marks the limiting velocity between tendencies to cosmical aggregation and to cosmical dissociation $v = \frac{gt}{2}$, on Dec. 18, 1863 (*Proc. Am. Phil. Soc.*, ix, 284 7). On April 1, 1864 (*Ib.*, p. 357), I said: "Absolute rest is apparently an impossible condition of matter, for, to whatever extent the action of opposing forces may be relatively neutralized, the inconceivable rapidity of æthereal, planetary and stellar motions produces a constant change of place. * * * The sum of all the instantaneous energies is the same, whether the particle fall freely for any given time, or remain apparently at rest. All the potential energy which is transformed in one case into the actual energy of motion, in the other is counteracted by an equivalent and opposite actual energy of elasticity." On July 15, 1864 (*Ib.* p. 408), I suggested "that one of the most probable results of the rotation of the Earth with its atmosphere, in an æthereal medium, would be the production of two systems of oscillations, moving with the rapidity of light." In October and December, 1864, I presented to the American Philosophical Society the "Numerical relations of gravity and magnetism," for which the Society awarded its Magellanic gold medal, as furnishing "good reason to hope that by the application of mechanical laws to the several phases of the æthereal undulations which produce the phenomena of light, heat, electricity, polarity, aggregation and diffusion, we may obtain a clearer understanding, not only of all the meteorological changes, but also of seismic tremors, crystallization, stratification, chemical action, and general morphology," (*Ib.*, p. 439). On Sept. 21, 1866 (*Op. cit.*, x, 269), I gave my first indication of the photodynamic importance of Earth's situation at the centre of the belt of greatest condensation, and on April 2, 1869 (*Ib.*, xi, 106-7), I showed that Sun's nascent or dissociative velocity is the velocity of light.

201. *Nitrogen and the Perissads.*

If Newton's belief that the inter-stellar æther is an expanded, universal atmosphere, is true, it seems likely that the two principal constituent gases of our own atmosphere may be everywhere as abundant, relatively, as they are within the reach of our immediate observation. Even if this is not the case, we may reasonably look for some special mathematical evidences of the importance of two gases which have so wide a local diffusion, and which have so large a sway in chemical combination and in organic

growth. In Note 54 I showed that a large number of the elements contain either 7 or 8, as one of the factors of the integers which most nearly represent their atomicity according to the hypotheses of Dalton and Prout. These two numbers denote respectively the simplest phyllotactic sub-multiples of N and O. If we use the former as a divisor of the perissads, treating the remainders as in the foregoing notes, we get the following results :

$$D_p = 7.$$

	Coefficient of D.	Remainder.	Log. R.
Li	1	+ .0073	$\overline{3}.8633229$
Na	3	+ 1.998	.3005955
K	6	— 2.981	.4743620
Cs	19	— .417	$\overline{1}.6201360$
Fl	3	— 2.016	.3044905
Cl	5	+ .370	$\overline{1}.5682017$
Br	11	+ 2.768	.4421661
I	18	+ .557	$\overline{1}.7458552$
Ag	15	+ 2.675	— .4273238
Tl	29	+ .715	$\overline{1}.8543060$
Rb	12	+ 1.251	.0972573
Sum of Monatomic logarithms			$\overline{2}.6980170$
N	2	+ .021	$\overline{2}.3222193$
P	4	+ 2.958	.4709982
As	11	— 2.082	.3184807
Sb	17	+ .955	$\overline{1}.9800034$
Bi	30	— 2.477	.3939260
Au	28	+ .155	$\overline{1}.1903317$
Bo	2	— 3.059	.4855795
Ta	26	+ .144	$\overline{1}.1583625$
V	7	+ 2.256	.3533391
Sum of Tri- or Pentavalent logarithms			$\overline{2}.6732404$

Log. of Monatomic probability ; $\log. 1.75^{11} - \overline{2}.6980170 = 3.9754016$
 “ Tri- and Pentavalent “ ; “ $1.75^9 - \overline{2}.6732404 = 3.5141020$
 “ Total Perissad 7.4895036

202. Oxygen and the Artiads.

If we use 8 as a divisor of the Artiads, we get the following results :

$$D_a = 8.$$

	Coefficient of D.	Remainder.	Log. R.
O	2	— .0367	$\overline{2}.5646661$
S	4	— .016	$\overline{2}.2041200$
Se	10	— 1.203	.0802656
Te	16	— .040	$\overline{2}.6020600$
Mg	3	— .041	$\overline{2}.6127839$

	Coefficient of D.	Remainder.	Log. R.
Ca	5	— .010	2.0000000
Sr	11	— .626	1.7965743
Ba	17	+ .763	1.8825245
C	1	+ 3.9736	.5991841
Si	4	— 3.805	.5803547
Ti	6	+ 1.846	.2662317
Zr	11	— 1.367	.1357685
Sn	15	— 2.302	.3621053
Hg	25	— .288	1.4593925
Mo	12	— .473	1.6748611
W	23	— .390	1.5910646
U	30	— 1.518	.1812718

Sum of Di- and Tetratomic logarithms

8.5932287

Gl	1	+ 1.085	.0354297
Al	3	+ 3.009	.4784222
Sc	5	+ 3.980	.5998831
Cr	7	— 3.991	.6010817
Fe	7	— .087	2.9395192
Ga	9	— 3.146	.4977587
In	14	+ 1.398	.1455072
Zn	8	+ .905	1.9566486
Cd	14	— .230	1.8617278
Mn	7	— 2.094	.3209767
Ni	7	+ 1.928	.2851070
Co	7	+ 2.887	.4604468
Cu	8	— .827	1.9175055
Pb	26	— 1.529	.1844075
Ru	13	+ .217	1.3364597
Rd	13	+ .055	2.7403627
Pd	13	+ 1.737	.2397998
Ir	24	+ .651	1.8135810
Pt	24	+ 2.415	.3829171
Os	25	— 1.506	.1778250
Yt	11	+ 1.816	.2591158
Ce	18	+ 3.576	.5533975
La	17	+ 2.526	.4024333
Di	18	+ .573	1.7581546
Er	21	— 2.109	.3240766
Th	29	+ 1.414	.1504494
Yb	22	— 3.239	.5104109

Sum of Metallic logarithms,

2.4334051

Log. of Di- and Tetratomic probability; $\log. 2^{17} = 8.5932287 = 12.5242813$			
“ Metallic	“	“ $2^{27} = 2.4334051 = 5.6944049$	
“ Total Artiad	“		18.2186862
“ Aggregate, Per. and Art.	“		25.7081898
“ Mean	“		.4016920

By reference to Notes 181 and 183, it will be seen that the aggregate probability of atmospheric phyllotactic influence is more than 37.6 times as great as that of simple phyllotactic influence, more than 4510 times as great as that of Gerber's divisors, more than 210363 times as great as that of the hydrogen divisor, or more than 2,083,840,000,000,000 times as great as that of the first surd divisor.

203. *Precipitability.*

The Philosophical Magazine for March 1882, contains two papers, one by Mills and Bicket, the other by Mills and Hunt, on chemical equivalence, as estimated by "equivalent precipitability of sulphates, by sodic carbonate, from an aqueous solution." Among the conclusions which they have drawn from their work the following seem to be especially noteworthy:—1. Precipitability is a linear function of mass. 2. There is some evidence that the precipitabilities of the commixed and separate sulphates are mathematically related in a simple manner. 3. Within moderate limits, precipitation is not traceably affected by temperature. 4. Two elements belong to the same group when, in saline solutions of identical genus, they may be equally precipitable. The simplicity and character of these conclusions are such as to suggest æthereal influence, a suggestion which is strengthened by the final equation, $\nu = \delta = .3819$; ν and δ being, respectively, the ratio of precipitability to the quantity of nickelous and cadmic sulphate taken. The ratio is the same, to the fourth decimal place, as the first surd divisor in extreme and mean ratio, .381966, thus indicating a beginning of phyllotactic tendency which is very satisfactory.

204. *Electrical Conductivity of Gases.*

Edlund (*P. Mag.* [5] xiii, 201), cites the experiments of Edm. Becquerel (*Ann. de Ch. et de Ph.* [3] xxxix, 377), showing "that gases begin to be conductors when heated to the temperature of redness, after which their conductivity increases in proportion as the temperature rises above that point," the conductivity increasing as the density of the gas diminishes. This approach to the æthereal condition is also an approach to the fundamental æthereal *vis viva*, which is shown by the identity of velocity in the propagation of luminous undulations, the electrical "ratio," and the gravitating reactions of stellar rotation.

205. *Ratio of Æthereal Elasticity to Density.*

In my first approximation to the ratio between atmospheric and æthereal elasticities (*Proc. Amer. Phil. Soc.*, ix, 440), I followed Herschel, in supposing that the velocity of light, in the interstellar spaces, is uniform, and that, consequently, the elasticity of free æther varies directly as its density. The same conclusion would follow from Newton's views (See Note 200), and is involved in Edlund's discussions of the relations of electricity to heat. Every additional evidence of harmonic relations, that is brought to light through the application of the laws of gaseous elasticity to the kinetic æther, is also an additional evidence of the truth of the hypothesis that all physical energy is transmitted by means of a universally diffused elastic medium.

206. *Spectrum of Lightning.*

Schuster (*P. Mag.*, [5] vii, 319), gives some of his measurements, at different times, of lines or bands in the lightning-spectrum, comparing the means with two measurements which Vogel had deduced from his own observations. The harmonic character of the lines is clearly shown by the following tables. The value of a , in the harmonic divisors, is .01578.*

Schuster.	Mean.	Vogel.	Harmonic.	Harmonic Divisors
5592			5592	1
5348				
5329	5334	5341	5339	$1 + 3 a$
5325				
5260			5260	$1 + 4 a$
5175				
5193	5182	5184	5183	$1 + 5 a$
5177				

207. *Torsion, Flexion and Magnetism.*

G. Wiedemann (*La Lumière Electrique*, vi, 90), in speaking of results obtained by the torsion of wires and the flexure of rods, says that the phenomena correspond so closely with those of magnetism that the words "torsion" and "magnetism" are almost always interchangeable. This is a further illustration of the identity of fundamental *vis viva*, which was spoken of in Note 204, and which is especially exemplified by the application of Coulomb's torsional coefficient to solar rotation (Note 162). My earliest "numerical relations of gravity and magnetism" (*Proc. Amer. Phil. Soc.*, ix, 355—60, 425—40, *et al.*) were based upon the mechanical consequences of rotation in an elastic medium.

208. *Phyllotaxy and Atomic Heat.*

The constant product of atomic weight by specific heat, 6, which is indicated by the law of Dulong and Petit, is equivalent to the continued product of the first three numbers of the phyllotactic series $1 \times 2 \times 3$. Were

this an isolated fact, little importance could be attached to it, but when we bring it to the test of mathematical probability, it becomes suggestive of relations which may, perhaps, lead to important discoveries. In the following comparison D indicates the estimate of atomic heat which deviates least from 6 (*Meyer, Modernen Theor. d. Chem.* Ed. of 1880, 90, 106); δ_1 , the ratio of deviation from exact correspondence with the theoretical value; C, the observed multiples and values of the perissad and artiad divisors (Notes 201—2); δ_2 , their deviation from the theoretical values:

	D	δ_1	C	δ_2
Li	6.6	.1000	1×7.0	.0000
Na	6.7	.1167	3×7.7	.1000
K	6.5	.0833	6×6.5	.0714
Fl	5.	.1667	3×6.3	.1000
Cl	6.4	.0667	5×7.1	.0143
Br	6.7	.1167	11×7.3	.0429
I	6.8	.1333	18×7.0	.0000
Ag	6.	.0000	15×7.2	.0286
Tl	6.8	.1333	29×7.0	.0000
Rb	6.4	.0667	12×7.1	.0143
N	5.	.1667	2×7.0	.0000
P	5.9	.0167	4×7.7	.0429
As	6.1	.0167	10×7.5	.0714
Sb	6.	.0000	17×7.1	.0143
Bi	6.5	.0833	30×6.9	.0143
Au	6.4	.0667	28×7.0	.0000
Bo	5.5	.0833	2×5.5	.0714
O	4.	.3333	2×8.0	.0000
S	5.7	.0500	4×8.0	.0000
Se	6.	.0000	10×7.9	.0125
Te	6.	.0000	16×8.0	.0000
Mg	6.	.0000	3×8.0	.0000
Ca	6.8	.1333	5×8.0	.0000
Sr	6.4	.0667	11×7.9	.0125
Ba	6.4	.0667	17×8.0	.0000
C	5.5	.0833	1×12.0	.5000
Si	5.7	.0500	4×7.0	.1250
Ti	6.4	.0667	6×8.3	.0375
Zr	6.	.0000	11×8.1	.0125
Sn	6.5	.0833	15×7.8	.0250
Hg	6.3	.0500	25×8.0	.0000
Mo	6.9	.0167	12×8.0	.0000
W	6.1	.0167	30×7.9	.0125
Gl	5.6	.1000	1×9.1	.1375
Al	5.8	.0333	3×9.0	.1250
Cr	6.4	.0667	7×7.4	.0750

	D	δ_1	C	δ_2
Fe	6.3	.0500	7×8.0	.0000
Ga	5.5	.0833	9×7.7	.0375
In	6.5	.0833	14×8.1	.0125
Zn	6.1	.0167	18×8.1	.0125
Cd	6.	.0000	14×8.0	.0000
Mn	6.7	.1167	7×7.7	.0375
Ni	6.4	.0667	7×8.3	-.0375
Co	6.3	.0500	7×8.4	.0500
Cu	6.	.0000	8×7.9	.0125
Pb	6.3	.0500	26×7.9	.0125
Ru	6.3	.0500	13×8.0	.0000
Rd	6.	.0000	13×8.0	.0000
Pd	6.3	.0500	13×8.1	.0125
Ir	6.3	.0500	24×8.0	.0000
Pt	6.3	.0500	24×8.1	.0125
Os	6.2	.0333	25×7.9	.0125
Ce	6.3	.0500	18×7.8	.0250
La	6.2	.0333	17×8.1	.0125
Di	6.4	.0667	18×8.0	.0000

This comparison shows that the general deviations from Dulong and Petit's law, while they are of the same order of magnitude, are much greater than the deviations from the perissad and artiad divisors.

209. *Secondary Character of Perissad Phyllotaxy.*

Although the fractions which are formed by successive approximations to the surd divisors represent phyllotactic dextro- and lævo-gyration, other series of a higher order may spring from greater initial differences. If we skip the first even number, we get the series 1, 3, 4, 7, 11, 18, etc. Hence we see that the fundamental perissad and artiad divisors both start from the phyllotactic number which most nearly represents the first surd divisor, 3, and are formed by adding the next artiad number for the perissad divisor, and the next perissad number for the artiad divisor. The coefficient of atomic heat is also formed from the same representative of division in extreme and mean ratio by taking its simplest artiad multiple, 2×3 .

210. *Comparison of Probabilities.*

In looking more closely into the deviations which are given in Note 208, we find the following indications of superiority in the perissad and artiad divisors :

1. The approximation of the observed values within .05 of the theoretical values occurs 19 times in my columns, and only 9 times in those of Dulong and Petit.

2. The average deviations are, $\delta_1 = .0642$; $\delta_2 = .0344$.

3. The sums of the logarithms of the reciprocals of deviation, which indicate the aggregate relative probabilities of normal influence, are : 70.4555173 ; 89.2627807.

4. The ratio of aggregate probabilities is, therefore, $P_1 : P_2 :: 1 : 64159 (10)^{14}$. The ratio of mean probabilities, $P^{\frac{1}{5}}$, is $p_1 : p_2 :: 1 : 2.1976$.

5. Testing the hydrogen unit in a like way, I find the average deviation, $\delta_3 = .0024$; $\Sigma \log (1 \div \delta_3) = 152.5459742$; log. relative probability, taking P_D as the unit, 82.0904569. This gives, $P_3 = 12315 (10)^{78}$; $p_3 = 31.0852$.

6. In accordance with the principle of least squares, these values of p should be reduced inversely as the fundamental divisors. This gives $p_1 : p_2 : p_3 : p_4 :: 1 : 1.71 : 4.44 : 5.08$; $\log (P_2 \div P_1) = 12.8482860$; $\log (P_3 \div P_1) = 35.6100415$; $\log (P_4 \div P_1) = 38.8306765$; p_4 and P_4 being respectively, the mean and aggregate probabilities of the phyllotactic divisors (Note 181). The corresponding mean relative probabilities for S_1 , S_2 , and Gerber's divisors are respectively, 2.78, 2.45, 4.72.

211. *Suggestions for Further Investigation.*

The ratio between p_1 and p_3 , in the foregoing note, has been gradually diminished by successive approximations, and by making allowance for theoretical considerations, which have seemed to justify the adoption of some exact multiple or submultiple of an atomic weight which had been previously accepted. The ratio between p_1 and p_2 , favorable as it already is for the latter, is based upon a comparison of the latest revision of the atomic heats with the first crude application of the perissad and artiad divisors. If Dulong and Petit's law is entitled to great weight in determinations of atomicity, a still stronger claim may be urged in behalf of divisors which have a mean probability that is more than 70 per cent. greater. If Dumas's proposed modification of Prout's hypothesis were applied to Si and Cr, their atomicities would be very closely represented by $\frac{7}{2} \times 8$ and $\frac{1}{2} \times 8$; P is very nearly $\frac{5}{3} \times 8 \times 7$, or very nearly $\frac{4}{3}$ of the monatomic phyllotactic divisor ; Na is about 30 times the monatomic divisor or 2×11 , 11 being the phyllotactic number which follows 7 in the secondary series (Note 209) ; Bo is 7.018 times its proper phyllotactic divisor. Si, Cr, Na and P may, perhaps, have tendencies towards the opposite group, perissad or artiad, the investigation of which may throw light upon the beginnings of valency.

212. *Chemical Electricity.*

Davy's discovery of potassium laid the foundation of electrolysis, introducing polarity as an important modifier of chemical attraction. The attractions and repulsions of Sir William Thomson's hypothetical vortex-atoms involve gyroscopic tendencies to maintain uniform planes of rotation, which must aid the normal arrangements of æthereal particles (*Proc. Am. Phil. Soc.*, xii, 408) in the determination of axial and polar, centrifugal and centripetal relations. Hence arise various combinations of

motion and tendencies to motion, which are obedient to simple mechanical laws, and which give rise to the different classes of radiodynamic phenomena which we call gravitating, electric, magnetic, thermal, chemical, etc. In consequence of the universality of motion, which seems to make absolute equilibrium an absolute impossibility, the tendencies to division in extreme and mean ratio are never repeated in the same exact plane, but they partake of a more or less intricate spiral character, such as is uniformly shown in vegetable growth. The comparative relative stability of axes, even in ultimate molecules and atoms, must produce æthereal oscillations which are parallel to the axis, as well as those which are radial and tangential (*Op. cit.*, ix, 408), giving rise to solenoidal currents, such as are assumed in Ampère's hypothesis.

213. *Earth's "Pulsation Period,"*

Proctor (*Contemporary Rev.*, March, 1882, p. 479), speaks of "the time when the Earth's rotation began to approach to synchronism with her pulsation period" or "the period of vibration of her mass after any impulse (affecting the whole Earth) had been received from without. The Earth would as certainly have had such a pulsation period as the vibrating substance of a bell has." This admission is interesting as an evidence of increasing recognition of the truths which are involved in Herschel's doctrine of nebular elasticity or quasi-elasticity, and which are the groundwork of all my harmonic researches. Proctor, however, in trying to explain the supposed retardation of Earth's rotation, overlooks the more than three hundred-fold acceleration which Laplace's hypothesis would require.

214. *The "Reproach" of Thermodynamics.*

The hypothesis that stellar systems are cooling, condensing and giving out heat, imparting their *vis viva* to the luminiferous æther without receiving anything in return, and that, consequently, all things are tending to ultimate physical stagnation and universal death, is so unphilosophical and altogether unsatisfactory as to show that some important element must have been overlooked. If we were granted infinite elasticity, or a medium acting under elastic laws but without density, Laplace's supposed instantaneous transmission of gravitating action might be represented by well-known physical formulæ. In other words, if we could conceive of a material medium endowed with qualities which are not material, some of the difficulties of pure materialism would be removed. What name could be given to such a medium, but spirit? Spiritual, conscious, "upholding" and controlling power is conceivable; without such a conception, the most important of all phenomena are wholly inexplicable. Any hypothesis that an unconscious universe could ever have wound itself up like a clock, is childish; the belief that, after having wound itself up, it would allow itself to run down without winding itself up again is more childish still. The confession that we can see no escape from final stag-

nation imposes no restraint on the universe ; it is only a confession of our own shortsightedness. He who sees the necessity of a Wise, Everlasting and Almighty Omnipresence, also sees that the present order of things must continue as long as its Ruler wills. He who sees that the Omnipresent Power acts "in ways which may be represented by harmonic or cyclical undulations in an elastic medium," also sees that more is implied in the equality of elastic action and reaction than has yet been fathomed by the sounding line of the most skillful analysis.

215. *Tides.*

The danger of hasty generalizations from investigations which are necessarily of a partial character, is well illustrated by the various speculations which have been set forth about tidal action. The equilibrium-hypothesis and each of the dynamic hypotheses have severally considered important relations and interactions between the disturbed and disturbing bodies, but the incompleteness of them all is shown by our inability yet to explain some of the phenomena which are of daily occurrence, as well as by our complete ignorance as to the normal position of the tidal crests. Bernouilli, and Laplace for certain mean depths of ocean, assumed that it should be high water under the moon ; Laplace for other depths, Delaunay and Airy have given satisfactory evidences of tendencies to high tide when the moon is in the horizon ; sailors have a prevalent belief that the high water, in mid-ocean, lags about three hours behind the moon ; many mathematicians think that either friction or inertia may produce such lagging, but it has never been shown that there is any tidal friction, or that inertia can delay any normal tidal action. Some of the most satisfactory results have been reached through considerations of the elasticity which is involved in wave-propagation, but the inter-molecular elasticity, the extent to which the several particles of water are free to fall towards or recede from the attracting body, and the variations of weight consequent on variations of gravitating tendency, have not been sufficiently studied.

216. *Barometric Analogy.*

Fortunately, upon at least one of the foregoing points, we can ask nature a simple question, to which she gives a satisfactory answer. Is there any evidence of tidal disturbance of weight ? Yes, in the daily fluctuations of the barometer. They are certainly tidal, even if we fail to see in them any likeness to the ocean tides. The air, which is heated and expanded by the sun's rays, is carried forward by the earth, in its orbital revolution and daily rotation, with a continual tendency of each particle to maintain the instantaneous direction of its motion. This tendency is represented, not by the simple momentum of the particles, but by their *vis viva*, and is accompanied by gravitating tendencies, which are sometimes antagonistic and sometimes co-operative, towards the earth and towards the sun. Their own elasticity concurs with the elasticity of any intervening me-

dium, in adjusting their relative positions to the ever-varying requirements of equilibrium, and causing harmonic oscillations which are easily traceable by means of systematic barometric observations. There can be no friction, provided the adjustments are made by the simple approach or separation of particles, and such appears to be the case. In the most thorough series of observations that has been published for any station near the equator, the harmonic oscillations are of the simplest character conceivable, representing the quarter-daily sums of the instantaneous tendencies and the changes in atmospheric weight so accurately as to give an estimate of Sun's distance, which differs by less than one-half of one per cent. from the latest astronomical estimates (*Proc. Am. Ph. Soc.*, ix, 287; x, 375-6, *foot note*.)

217. *Ratio of Tidal Adjustments.*

Sir William Thomson has found a partial solution of the theoretical requirements of terrestrial rigidity, in his theory of vortex atoms. Perhaps the solution may be completed by supposing an intermolecular elasticity which is greater than that of glass, instead of a rigidity which is greater than that of glass. The influence of atmospheric pressure on the height of ocean tides, which has been noticed by many observers, suggests the likelihood that the whole mass of the earth may contribute to the adjustments of equilibrium which satisfy tidal tendencies. If that is the case, the entire change which would be required in the distance between any two molecular centres is less than $\frac{1}{4000000}$ of their mean distance, even at the spring tides, when the sun and moon combine their disturbing energies. The whole adjustment might be accomplished, through æthereal elasticity, in less than $\frac{1}{20}$ of a second, but it only needs to be accomplished four times in about 25 hours.

218. *Summation of Tendencies.*

The triumphs of calculus spring from the fact that its differentials represent only tendencies and its integrals are summations of tendencies. No integration or series of integrations can be rightly looked upon as conclusive, unless it has been extended to all the tendencies which can have any bearing upon the problem which we are examining. Nothing is more certain than mathematics, except our knowledge of our own spiritual existence and faculties. Neither in mathematics nor in psychology, however, is it safe to assign any value to our results beyond their necessary relations to the data from which they were obtained. Delaunay's hypothesis of tidal friction undoubtedly follows from his postulates, and if we accept it, we may be satisfied with the explanation which it gives of apparent lunar retardation, but his postulates are not all axiomatic; they do not cover the whole ground; and the errors in the lunar tables may spring from some portion of a cycle of mutually compensating perturbations. The tidal tendencies are towards accelerated rotation in two of the quad-

rants and towards retarded rotation in the other two, the sum of the accelerating being exactly equal to the sum of the retarding tendencies. No evidence has ever been adduced of any actual lagging of the water to maintain the normal position of the tidal crests relatively to the moon. There are many reasons for believing that the apparent westward motion, with a mean equatorial velocity of 1000 miles an hour, is only a motion of form, maintained by the combined influences of intermolecular elasticity, atomic elasticity or quasi-elasticity, variations of pressure on account of varying attraction, and such wave propagation as may be needed for the adjustment of opposite meridional and horizontal, static and dynamic tendencies. The adjustment may be brought about, as I have shown in Note 217, without any frictional diminution of the speed of rotation.

219. *The Moon and the Chief Planetary Belt.*

The importance of Earth's position, at the centre of the belt of greatest condensation, is further shown by the harmonic reactions between the Jupiter-Saturnian belt and Earth, with its satellite. The shortening of rotation-period which would represent a nebular contraction of Sun from Jupiter's to Earth's mean locus, corresponds to the shortening which would represent a contraction from Moon's semi-axis major to Laplace's terrestrial limit; the ratio between Moon's synodic and sidereal periods corresponds to the ratio between the locus of Saturn's incipient subsidence (secular aphelion) and axis-major. The time of rotation, in an expanding or contracting nebula, varies inversely as the square of radius:

$$(\rho_5 \div \rho_s)^2 = 5.2028^2 = 27.06912.$$

$$\text{Sidereal month} \div \text{day} = 27.32166.$$

$$\text{Synodic} \div \text{sidereal month} = 1.08087.$$

$$\text{Saturn's sec. aph.} \div \text{mean}^* = 1.08433.$$

220. *Stability of Rotation-Periods.*

The relations of stellar rotation to oscillations which are propagated with the velocity of light, the relations of primary planetary rotation to planetary revolution, the relations of molecular rotation to electric, magnetic and tidal phenomena, the constancy of tendencies to harmonic oscillation, the confirmation of nebular theories which is afforded by the foregoing note, and, the principle that no change in the *vis viva* of a system can take place without foreign action, all indicate a stability of rotation which is inconsistent with the hypothesis of tidal friction. Moreover, the closeness of accordance between the mean daily thermal and hygrometric adjustments of elasticity and the tidal variations of atmospheric pressure (*Proc. Am. Ph. Soc.*, ix, 284-6, 291-3, 346-8), an accordance which is also shown in the lunar-monthly barometric tides (*Ib.*, 395-9; *Proc. Roy. Soc. xiii*, 329-33), furnishes additional grounds for believing that rotation is only modified revolution, that its period is deter-

* According to Stockwell.

mined by a summation of all the tendencies to revolution which bear upon each and all the molecules of the rotating body, and that tidal variations of weight or pressure are as important in earth- and ocean-tides as in atmospheric tides.

221. "*There is much Virtue in If.*"

Some extracts from a lecture by Dr. Ball, the Astronomer Royal of Ireland, have lately been largely copied by the newspapers. They contain a statement that the moon was once only 40,000 miles away, and that it thus acted as a geological engine of transcendent power. The statement is somewhat qualified by the proviso that if the present tides are three feet, and if the early tides were 216 times their present amount, then it is plain that the ancient tides must have been 648 feet. This qualification is not sufficient, and it is misleading, because it will be generally understood as covering all the points about which there is any uncertainty. Science in its claims of exactness, cannot afford to hazard any claims which can be easily refuted. It is true that there are many astronomers who believe that Delaunay's views are correct, but there are probably few who think that they have been conclusively demonstrated. If the moon pulls the ocean-waters around the earth, in a direction opposite to its daily rotation, at the rate of a thousand miles an hour, or at any less rate; if the friction, which would result from such a pull, is not compensated in some way which is not yet fully known; if there is a bulge of tidal water which cannot fully keep up with the moon, and which, by its attraction on the moon, tends to retard its orbital velocity; if all the mathematical conclusions which it seems reasonable to draw from such supposed retardation are correct, and if the "reproach" of thermodynamics must be accepted without qualification, the moon may be receding from the earth.

222. *Weakness of the Postulates.*

In examining the provisos of the foregoing note, we find :—In the first place, no tidal currents have ever been observed which indicate a lagging tendency in ocean waters. Secondly, there is no evidence whatever to show that the earth's rotation has been retarded by friction. Thirdly, there is no evidence to show that the moon's orbital motion has been retarded by the ocean tides. Fourthly, the number of elements which must enter into any calculation of planetary disturbances is so great that no prudent mathematician ever looks for more than an approximation to such results as he desires. Fifthly, the difficulties which are encountered in trying to explain irregularities of orbital motion, are vastly enhanced when we come to deal with the complicated tendencies of planetary rotation. Sixthly, there is as much reason to believe that the moon may be gradually falling to the earth, as there is to believe that the earth may be gradually falling to the sun. Seventhly, the accelerating and retarding tendencies of æthereal elasticity and resistance are but little understood. Eighthly, all of the possible compensatory adjustments, to which I have

referred in foregoing notes, should be thoroughly investigated before forming any conclusive opinion respecting Delaunay's hypothesis. Ninthly, even after such investigation, the remembrance of other possible unknown influences should prevent anything like dogmatical assertion.

223. *The "Ifs" of Elasticity.*

I shall not shrink from any criticism such as is implied in the following "retort courteous": If there is a universal æthereal medium; if it is endowed with an elasticity somewhat like that of gases; if its velocity of wave-propagation can be expressed by the ordinary formula of relation between elasticity and density; if the laws of harmonic vibration in elastic media, which have been mathematically deduced, are correct; if the æthereal *vis viva* can be shared with chemical atoms and cosmical masses; if nebular "subsidence" has been governed by the laws of gravitation; if all kinds of energy are simple functions of mass and velocity, and "if all the mathematical conclusions which it seems reasonable to draw from" these hypotheses are correct, the general postulate that "all physical phenomena are due to an Omnipresent Power, acting in ways which may be represented by harmonic or cyclical undulations in an elastic medium" may be accepted as a good working hypothesis.

224. *Acceptance of the Issue.*

These provisos cover the whole ground, as fully as I could wish. I have never claimed, nor have I believed, that any scientific thesis can be freed from the limitations which are involved in its fundamental assumptions. While I fully believe in the impossibility of anything acting except where it is, in the existence of a universal elastic medium which is governed by radiodynamic and harmonic laws, and in the uniformity of physical force, I am well aware that they are incapable of mathematical demonstration and I have repeatedly acknowledged that the nebular and æthereal hypotheses have no scientific value beyond such helpful coördination of phenomena as they may furnish. The tidal "ifs" are mere assumptions, adduced in order to account for an apparent retardation which is altogether problematical and which, if it should prove to be real, may be followed by an equivalent acceleration; the elastic "ifs" are all intrinsically probable, and instead of having been assumed for a special purpose they represent simple and natural generalizations from a wide range of independent physical phenomena. The tidal ifs are like Bacon's "barren virgins;" the elastic ifs have already led to the discovery of a vast number of natural harmonies and the field for further like discovery widens so rapidly that every physical atom seems to contribute its individual melody, to the ever-resounding and ever-changing choral strains which constitute the music of the spheres. Although centripetal and centrifugal activities may be expressed by identical formulæ, it is difficult, if not impossible, to form any definite conception of attracting pulls. Elastic thrusts are exemplified by every breath that we draw, every object that we see, every sound that

we hear, and Anderssohn* has experimentally shown that they can adequately represent all varieties of gravitating and electromagnetic phenomena.

225. *A Scientific Statement of the Tidal Problems.*

The "Astronomy for Schools and Colleges," by Newcomb and Holden (Ed. of 1879, p. 167), speaks with true scientific caution, as follows:—"The theory of the tides offers very complicated problems, which have taxed the powers of mathematicians for several generations. These problems are in their elements less simple than those presented by the motions of the planets, owing to the number of disturbing circumstances which enter into them. The various depths of the ocean at different points, the friction of the water, its momentum when it is once in motion, the effect of the coast-lines, have all to be taken into account. These quantities are so far from being exactly known that the theory of the tides can be expressed only by some general principles which do not suffice to enable us to predict them for any given place."

226. *Cometary Spectra.*

The uncertainties of measurement and the harmonic indications which are given in spite of those uncertainties may be illustrated by comparing observations of like objects by different reporters. Tacchini gives (*Ann. de Chim. et de Phys.*, *xxv*, 286) measurements of the spectral lines in comet *b*, 1881, which correspond satisfactorily with lines in Hesselber's carbon spectrum. The harmonic accordance is equally satisfactory.

Harmonic.	Tacchini.
$37527.7 \div 68 = 551.9$	552.1
$37527.7 \div 73 = 514.1$	514.1
$37527.7 \div 81 = 463.2$	463.1

Thollon (*Ib.*, 287-8) compares the same spectrum with three different spectra of carbon compounds, viz: A, electric arc, Jamin's lamp measurements made by M. Bigourdan; B, cyanogen, coil and condenser, Salet; C, blue flame of illuminating gas, Lecoq de Boisbaudran.

Harmonic.	Thollon.	A.	B.	C.
$31479 \div 56 = 562.1$	562	562.2	563.0	562.9
$31479 \div 61 = 516.0$	516	516.5	516.3	516.1
$31479 \div 67 = 469.9$	470	470.4	470.0	470.6†

The harmonic divisors for Tacchini's measurements are sums of successive or nearly successive phyllotactic numbers: $81 = 5 + 8 + 13 + 21 + 34$; $73 = 81 - 8$; $68 = 73 - 5 = 13 + 21 + 34$. In the harmonic divisors for Thollon's measurements, $56 = 7 \times 8 =$ product of the artiad and perissad divisors, and the middle line is an arithmetical mean between the other two.

* *Der Mechanik der Gravitation*, Breslau, 1874; *J. B. des Bres. Phys. Ver.*, 1881-2.

† Boisbaudran does not give this line, but he gives 473.8 and 467.5, the arithmetical mean being 470.65.

227. *Identity of Spectral Lines in Different Elements.*

Young (*Am. Jour. Sci.*, *xx*, 355) and Liveing and Dewar (*Proc. Roy. Soc.*, *xxii*, 225-31) have shown that many of the lines in different elementary spectra, which have been supposed to be identical, really differ slightly in refrangibility and can be separated by a sufficient increase of dispersive power in the trains of prisms. The number of separations which has already been effected makes it very doubtful whether any case of absolute coincidence can be found, where two elements are present in the spectral incandescence. This has been thought, by some, fatal to Lockyer's and Thalen's hypothesis that all the lines are modifications of a few basic lines. That such a generalization is too hasty, may be shown by the following considerations: 1. Atoms are continually subject to incommensurable, as well as to commensurable tendencies. 2. There are often various harmonic tendencies, which are simultaneously operative, the final harmonic adjustment being determined by the relative magnitude of the individual tendencies. 3. The well-known experiment of oscillating balls, suspended from a horizontal cord, shows that the cyclical vibrations are modified by each member of a harmonic group. 4. The slight fluctuations in the lines of the solar spectrum make it probable that there are similar fluctuations in chemical and cometary spectra. 5. This probability is increased by the differences of measurement which are made by different observers at different times. 6. Propositions 2 and 5 are both illustrated by the two harmonies which represent Tacchini's and Thollon's measurements (Note 226).

228. *Lithium Harmonies.*

Liveing and Dewar (*Proc. Roy. Soc.*, *xxx*, 93-9) have observed three lines in the spectrum of lithium (3913, 3984 and 4273), besides Boisbaudran's line, 4131.7. The harmonies are shown below.

Harmonic Divisors.	Harmonic Quotients.	Observed.
1	4273.02	4273
1 + 7 <i>a</i>	4132.78	4131.7
1 + 15 <i>a</i>	3983.37	3984
1 + 19 <i>a</i>	3912.65	3913.

The coefficient of the first addition to the harmonic divisor is the same as the perissad divisor and as Prout's coefficient of Li. The second and third additions are respectively the artiad divisor and $\frac{1}{2}$ the artiad divisor. The harmony is nearly as satisfactory, if we combine these lines with those which are given by Huggins (*see Proc. Am. Ph. Soc.*, *xvii*, 297).

Harmonic Divisors.	Harmonic Quotients.	Observed.
1	6107.37	6107.3
1 + 40 <i>a</i>	4796.64	4794.8
1 + 48 <i>a</i>	4599.23	4599.3
1 + 63 <i>a</i>	4269.74	4273.
1 + 70 <i>a</i>	4131.49	4131.7
1 + 78 <i>a</i>	3984.31	3984.
1 + 82 <i>a</i>	3914.50	3913.

The coefficients of a are, 5×8 , 6×8 , 9×7 , 10×7 , $10 \times 7 + 8$, $10 \times 7 + \frac{3}{2}$ of 8, being made up of multiples or sums of the phyllotactic numbers, 2, 3, 5 and 8, and the secondary phyllotactic number, 7.

229. *Relations of Central Force to Thermal Constants.*

I have shown (*Proc. A. P. S.*, *xiv*, 651) that the ratio of heat under constant volume to heat under constant pressure, as deduced from purely theoretical considerations, is $\pi^2 + 4 : 2\pi^2$, or 1 : 1.4232. The elements for computing this ratio are : 1, the synchronism of oscillations, under the action of central forces, in all orbits which have the same major axis ; 2, the kinetic theory of gases, which supposes that all the paths of clashing particles are rectilinear, and therefore in orbits of unitary eccentricity, one extremity of each path corresponding with the centre of a synchronous circle ; 3, the consequent ratio of mean rectilinear *vis viva*, or mean *vis viva* of constant gaseous pressure, to synchronous mean circular *vis viva*, or mean *vis viva* of constant volume ; 4, the thermodynamic doctrine that equal quantities of heat correspond to equal increments of *vis viva* and to equal increments of temperature ; 5, the proportionality of mean *vis viva* to mean distance of projection against uniform resistance ; 6, the determination of the radial locus at which the mean velocity of linear oscillation, or of mean gaseous pressure, would be acquired both in centrifugal and in centripetal motion. This theoretical determination of the ratio of specific heats proceeds on the hypothesis of Boscovich, that central forces continue to act, at all distances from the centre, with accelerations which vary inversely as the square of the distance. There are many reasons for believing that this law does not hold, even in the æthereal condition, within the radius of inertial aggregation, and it seems likely that careful experiments may bring to light many kinds of deviation from the theoretical value, the study of which will greatly extend our knowledge of atomic and molecular structure. The most accurate experimental determinations of the ratio that have been published hitherto seem to range between 1 : 1.4053 and 1 : 1.421. These values indicate an orbital eccentricity of from .9874 to .9985.

230. *Tests of Thermal Relations by Solar Mass and Distance.*

The estimates which I have hitherto made of the central energies of the solar system, from measurable tendencies to equilibrium between gravitating and explosive or centripetal and centrifugal energies (*Proc. A. P. S.*, *xii*, 392-4, *xix*, 354, *et al.*), have been based upon the supposition that all the calorimetric measurements were made under constant pressure. C. v. Than (*Abstr. in Jour. Chem. Soc.*, *March*, 1882, p. 265.) gives five estimates for the heat of combustion of H_2O , from which estimates of solar mass and distance may be deduced by the method of Note 16.

	Observers.	θ	ρ	m
At constant volume	Andrews,	33,880	92,760,000	331,500
	v. Than,	33,822	92,839,400	332,350
At constant pressure	J. Thomsen,	34,218	93,071,400	334,850
	Favre and Silbermann,	34,426	92,789,800	331,820
	Schuller and Wartha,	34,471	92,729,200	331,170

The observations were made respectively in 1848, 1881, 1873, 1852 and 1877. The corresponding molecular heats, as given by Naumann (see Note 16) for three of the above observers, differ slightly from $2 \times$ the above values of θ , the greatest difference being $\frac{2}{3}$ of one per cent. The mean values, if we allow equal weight to the present note and to Note 16, after making the proper correction in the observations at constant volume, are $\rho = 92,739,500$; $m = 331,280$. This value of ρ differs by less than $\frac{1}{333}$ of one per cent. from the mean of the combined results in Note 15 (92,737,100).

231. *Molecular Volume of Solids.*

E. Wilson (*Proc. Roy. Soc.*, xxxii, 457-91) discusses the relations of molecular volume to chemical constitution, furnishing new evidence of harmonic oscillation. He states the three following propositions, and thinks that his tables lend comparatively greater support to the third, while the first and second must, for the present, be considered more hypothetical:

(i.) When any number of *similar* atoms combine, the volume of the resulting molecule is equal to that of the uncombined atom.

(ii.) When dissimilar atoms combine, the volume assignable to each atom is some simple submultiple or aliquot part of its atomic volume, and the resultant molecular volume is the sum of those volumes.

(iii.) Every element in its various compounds is capable of assuming different volumes bearing a simple proportion to one another, such as 1 : 2, 1 : 3, 2 : 3, &c.

He also adduces evidence in support of Kopp's conjecture that elements may undergo different degrees of condensation in different radicles of the same compound, and he shows the agreement of his results with those which were obtained by Loschmidt from gaseous interdiffusion.

232. *Variability of Crystalline Angles.*

F. Pfaff (*Jour. Chem. Soc.*, June, 1881, *Abstr.* p. 356) has made a series of measurements, from which he concludes that the limits of admissible correction of measured angles by calculation from rational axial sections must be carried further than has hitherto been the case. W. H. Perkin (*Ib. Aug.*, 1881, 409-452), in discussing the isomeric acids obtained from coumarin and the ethers of hydride of salicyl, gives seven sets of crystalline measurements, with forty-nine comparisons of calculated and observed angles. Taking the range between the limits of observation, which are given in twenty-six of the comparisons, or the deviations of the observed

from the calculated values, in the other twenty-three comparisons, the variability is more than one per cent. in one-third of the whole number of measurements, viz : .155, .121, .067, .056, .055, .054, .046, .044, .021, .019, .018, .016, .016, .015, .012, .011. The mean variability of the forty-nine measurements is .017. These facts may have an important bearing upon many questions of radiodynamic probability, especially in regard to the adjustment of commensurable and incommensurable tendencies.

233. *Pressure.*

The experiments of Tresca and Spring, together with those of Crookes, Pictet and Cailletet, show that it is impossible to fix any boundaries between any two of the adjacent states of matter, æthereal, gaseous, liquid, solid, crystalline. J. and P. Curie (*Comptes rendus*, lxxxvi, lxxxvii) confirm Faraday's hypothesis that magnetized and dielectric bodies should tend to contract in the direction of the lines of force and to dilate at right angles to those lines, a tendency which, as I have shown,* is propagated with the velocity of light. They suppose that between the opposed faces of two contiguous layers of molecules there is a constant difference of tension, involving a condensation of electricity which depends on the distance between the two layers. By experiments with tourmaline and hemihedral crystals with inclined faces they are led to attach primary importance to the form of the molecules, the extremity which corresponds with the most acute solid angles being always negative on dilatation and positive on contraction. They deduce the following laws :

1. The two extremities of a tourmaline crystal develop quantities of electricity under pressure which are equal, but of opposite kind.
2. The quantity developed by a given increase of pressure is equal to that which is developed by an equal diminution of pressure, but of opposite kind.
3. This quantity is proportional to the variation of pressure, is independent of the length of the crystal, and for the same variation of pressure per unit of surface is proportional to the surface.

All of these results have an important bearing upon the old maxim that "nothing can act except where it is," and on Newton's consequent belief that the phenomena of gravitation can be more satisfactorily explained by æthereal pressure than by attracting pulls. They may also help to explain the formation and sublimation of heavy metallic elements, by the immense pressures to which the interior of condensing nebulae are subjected. Many of the aggregating and dissociative tendencies of "subsidence," of which my planetary harmonies have given abundant evidence, may be exemplified chemically as well as cosmically.

234. *Test of Harmonic Probability.*

I have endeavored, in my various physical papers, to collect facts, through the guidance of well-known laws, and to account for them by a

*See citations in Note 200.

reference to those laws, without introducing any new hypotheses. I have already compared various phyllotactic harmonies with other chemical hypotheses, and Note 232 furnishes data for extending the tests of mathematical probability. In my first paper on the harmonic interferences in the spectra of chemical elements (*Proc. A. P. S.*, xvii, 297-301) I examined the measured wave-lengths of 128 lines, in twenty-one different spectra. The greatest mean deviation of the measured lines in either spectrum from lines which are rigidly harmonic, is less than $\frac{1}{3}$ of one per cent., the mean deviation in the whole number of lines being less than $\frac{1}{13}$ of one per cent. The mean deviations in the several spectra are as follows: $\frac{1}{20}$, $\frac{1}{37}$, $\frac{2}{7}$, $\frac{1}{55}$, 0, $\frac{1}{14}$, $\frac{1}{14}$, $\frac{1}{175}$, $\frac{1}{11}$, $\frac{1}{16}$, $\frac{1}{103}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{412}$, $\frac{1}{37}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{29}$, $\frac{1}{25}$, $\frac{1}{66}$, $\frac{1}{13}$ of one per cent. The greatest deviation in any single line is one per cent., and there is only one line which has a deviation of more than $\frac{1}{2}$ of one per cent., which is only $\frac{1}{31}$ as great as the greatest deviation in Perkin's set of crystalline measurements, or less than $\frac{1}{3}$ as great as his mean variability. Later comparisons, of which Notes 226 and 228 may be taken as examples, show approximations which are still closer. The greatest deviation in Tacchini's cometary measurements is $\frac{1}{27}$ of .01, and the mean deviation $\frac{1}{51}$ of .01; the greatest deviation in Thollon's measurements is $\frac{1}{4899}$, and the mean deviation $\frac{1}{7740}$; the greatest deviation in the first lithium spectrum of Note 228 is $\frac{1}{3828}$, and the mean deviation $\frac{1}{7837}$; the greatest deviation in the second spectrum of the same note is $\frac{1}{1310}$, and the mean deviation $\frac{1}{4380}$.

235. *Spectrum of the Great Nebula in Orion.*

On the 7th of March, 1882, Huggins (*Am. Jour. Sci.*, [3] xxiii, 335) obtained a photograph of the spectrum of the nebula in Orion, with an exposure of 45 minutes. His former researches showed that the visible spectrum of gaseous nebulae contains four bright lines, 5005, 4957, and two of the hydrogen lines, β and γ . The photograph has also a strong line in the ultra-violet, at the position of λ 3730, or nearly so. Some of the harmonic relations of the lines are given in the following table:

Harmonic.	Observed.
$525405 \div 105 = 5003.86$	5005
$525405 \div 106 = 4956.65$	4957
$525405 \div 108 = 4864.86$	4861
$525405 \div 121 = 4342.19$	4340
$525405 \div 141 = 3726.28$	3730

The greatest deviation is $\frac{1}{10}$ of one per cent., and the mean deviation $\frac{1}{20}$ of one per cent.

236. *Magnetic Estimate of Æthereal Density.*

Newton's æthereal hypothesis, Faraday's electric hypothesis and my own numerical relations (See Note 200) are exemplified in the following combined harmonies: Let v_0 represent Earth's mean orbital velocity

which is due to Sun's attraction ; v_3 , corresponding magnetic component of circular orbital velocity which Earth would communicate to an æthereal particle ; θ_o , specific heat of water ; θ_3 , specific heat of typical gas ; d_o , density of Sun ; d_3 , density of Earth ; δ_o , mean density of æther in Earth's orbit under influence of Sun's attraction ; δ_3 , density of Earth's atmosphere at mean locus of magnetization. Then

$$v_3 : v_o :: \theta_3 : \theta_o \quad (1)$$

$$d_3 : d_o :: \delta_3 : \delta_o \quad (2)$$

The given values are, $v_o = 18.476$ m. ; $\theta_3 = .23773 \theta_o$; $d_o = .25491 d_3$. The required values are v_3 , δ_3 and δ_o . From (1) we find

$$v_3 = .23773 \times 18.476 = 4.3924 \text{ m.}$$

At Earth's equatorial surface, $\sqrt{gr} = 4.9073 = 1.1172 v_3$; the magnetic component of this velocity in Earth's orbital plane is $v_m = \cos. 23^\circ 28' \times 4.9073 = 4.501 = 1.0248 v_3$; the mean locus of magnetization is therefore, $1.0248^2 \times 20,923,654$ ft. from Earth's centre $= .05028 \times 20,923,654 = 1,051,985$ ft. from Earth's surface. According to Babinet's formula (*Smith's n Tables*, D, p. 68) the normal density of the air diminishes $\frac{1}{2}$ at the altitude

$$Z = 52494 \text{ ft.} \times \frac{30 - 15}{30 + 15} = 17498 \text{ ft.} = 1,051,985 \div 60.12$$

The atmospheric density at the locus of magnetization is, therefore $\delta_3 = 1 \div 260.12 = 1 \div 1,252,920,900,000,000,000$; the æthereal density, $\delta_o = \delta_3 \times .25491 = 1 \div 4,915,148,000,000,000,000$. The density of hydrogen is .0692, or, according to this estimate, $340,128,200,000,000,000 \delta_o$. This is 2.07 per cent. greater than the estimate which was based on the ratio of projectile gaseous energy to æthereal energy (Note 35). The significance of proportion (1) is increased by the cosmical relations of Joule's equivalent (*Proc. A. P. S.*, xix, 20). The agreement would be exact if we take $\rho_3 = 92,809,500$ miles.

237. "Subsidence" Estimate of Æthereal Density.

Subsidence towards the three chief centres of nebulousity, (Jupiter), condensation, (Earth), and nucleation, (Sun), should be influenced by æthereal harmonies. If we take the estimate of Sun's mass which satisfies the requirements of subsidence and oscillation (331776 ; Notes 5, 23, 91) and the British Nautical Almanac value for Earth's distance, measured in Sun's semi-diameters (214.45), $\rho_3 = 92,785,700$ miles ; the mean projectile locus of the chief centre of gravity in the system (c. g. Sun and Jupiter at mean perihelion) $= 1.018 r_o = r_c$; L_o (solar modulus of light ; Note 75) $= 474657 r_o = 465896 r_c$; the mean locus of magnetization, $l_3 = r_c \times \rho_3 \div L_o = 199.1555$ miles $= 1,051,541$ ft. $= 60.09498 \times 17498$. This gives, for the ratio of hydrogen density (δ_h) to æthereal density,

$$\delta_h = 334,280,400,000,000,000 \delta_o$$

which exceeds the estimate of Note 35 by less than $\frac{1}{2}$ of one per cent.

238. *Rotation Estimate of Æthereal Density.*

The hypothesis that hydrogen is the simplest known form of æthereal condensation and that all other chemical elements are condensed hydrogen, together with the theory that stellar rotation is due to æthereal harmonic oscillations (Notes 17, 34, 198 *et al.*), requires that the linear oscillations of the kinetic gaseous theory should be made circular, within the stellar nucleus. Since gaseous density varies inversely as volume, the æther-hydrogen hypothesis is satisfied by the proportion

$$\pi L_o^3 : r_o^3 :: \delta_h : \delta_o$$

∴

$$\delta_h = 335,961,800,000,000,000 \delta_o$$

which is $\frac{1}{2}$ of one per cent. greater than the estimate of the foregoing note.

239. *Æthereal Elasticity.*

The velocity of light (v_λ), according to the subsidence estimate, is $92,785,700 \div 497.827 = 186,381$ miles. The velocity of sound in hydrogen, according to Dulong, is 4163 ft. If we designate the ratio of elasticity to density ($e \div d$), for hydrogen and æther respectively, by ϵ_h and ϵ_o , the proportionality $v \propto \sqrt{\epsilon}$ gives

$$\epsilon_o : \epsilon_h :: (186381 \times 5280)^2 : 4163^2 :: 55,880,460,000 : 1$$

for the relative elasticities under the same density. If we adopt the rotation estimate of comparative density, we have

$$e_o : e_h :: 1 : 6,012,151$$

for the relative elasticities at normal density.

240. *Æthereal Density at Mean Planetary Loci.*

The æthereal density should be $\frac{1}{2}$ as great as at Sun's surface at $\frac{2}{3} L_o = 316,438 r_o = 1448.343^* \rho_3$. At any other locus, ρ_n , it should be $(\frac{1}{2})^n$, n being equivalent to $(\rho_n \div 1448.343 \rho_3)$. This gives, for the relative rotation estimate of æthereal density at Sun's surface and at the several planetary mean distances :

Sun	1.00000
Mercury	.99981
Venus	.99965
Earth	.99952
Mars	.99927
Jupiter	.99751
Saturn	.99544
Uranus	.99086
Neptune	.98573

* Allowing for rupturing centre of gravity of Sun and Jupiter.

241. *Validity of Estimates.*

All estimates of this character are, of course, only provisional, and they can claim no validity, as I have heretofore shown, beyond the accuracy with which they represent the data upon which they are based. That all the æthereal elements which I have considered are important, that they are more far-reaching than those which have been introduced into any like discussion of which I have any knowledge, that their influence has been rightly stated, and that they will contribute, by collation with Thomson's and other estimates, to a more satisfactory solution of many physical problems than is yet attainable, I fully and unhesitatingly believe. Sun's orbital motion, and questions connected with the retardations which change revolution into simple rotation, are among the considerations which seem likely to modify the values that are given in the five foregoing notes and in Note 35.

Note on the Aurora of April 16-17, 1882. By H. Carvill Lewis.

(Read before the American Philosophical Society, April 21, 1882.)

The aurora of Sunday evening, April 16-17, 1882, was probably one of the most remarkable, both as to beauty and scientific interest, that has been observed in this latitude. It is especially noteworthy on account of the brilliant *corona* which continued well defined for several hours, and whose apparent motion eastward, through space, could, therefore, be determined. Several other unusual features, such as an *auroral curtain*, and hyperbolic curves of light, were also displayed. The attendant solar and magnetic phenomena have also been of great importance in determining a theory of the aurora.

The aurora was noticed as soon as twilight had ended as a faint glow along the northern horizon. At 8.30 it was a low arch, probably not over 10 degrees high. It gradually rose higher, and left a dark segment below it. At 10 P. M. the arch was some 20 degrees high, and was constantly increasing in brilliancy. Bright short white acicular streamers now appeared in the north, and sometimes rose as high as 40 degrees. These occasionally assumed a reddish color, and were frequently wafted along the arch towards the west. The aurora now fluctuated greatly in brilliancy, sometimes nearly disappearing, and then flashing out brighter than ever.

At 11.15 the arch had become brighter and much longer, though still of low altitude. Bright acicular streamers were crowded closely together at the western end of the arch, while in the east a second arch was now formed. The auroral arch now began to rise rapidly. At 11.20 the upper arch was 40 degrees high. Long narrow streamers were rapidly forming over the whole northern sky, and were traversed from base to apex with